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تأثیر غیر خنثی بودن دینامیکی بر ناپایداری مدوله ای لیزر در پلاسمای داغ مغناطیده

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چکیده – ناپایداری مدولاسیون پالس کوتاه لیزر در پلاسمای داغ مغناطیده بررسی شده است. معادله ی انتشار غیر خطی لیزر با ساختار محدود طولی و عرضی در پلاسما حاصل شده است. تأثیر غیر خنثی بودن پلاسما بواسطه‌ی نیروی اثرگذار بر نرخ رشد ناپایداری مدولاسیون مطالعه شده است. نشان داده شده که افزایش شدت لیزر تا مقدار مشخصی سبب افزایش نرخ رشد گشته و در شدتهای بالا بواسطه‌ی تأثیرگذاری زیاد نیروی اثرگذار بر خروج الکترونها از ناحیه اندرکنش، سبب افت نرخ رشد می‌گردد. همچنین تأثیر پارامترهای اساسی اعم از میدان مغناطیسی خارجی، نوع قطبش و پهنای پالس بر ناپایداری تحقیق شده است.

کلید واژه- اندرکنش لیزر-پلاسما، معادله ی غیر خطی شرودینگر، ناپایداری مدولاسیون

Effect of Dynamical Non-Neutrality on the Modulational Instability of a Laser in Hot Magnetized Plasma

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Abstract- Modulational instability of short laser pulse in hot magnetized plasma is investigated. Nonlinear propagation equation of laser with finite longitudinal and transverse structure in plasma is obtained. Effect of plasma non-neutrality caused by the ponderomotive force on the modulational instability growth rate is studied. It is shown that increase in the intensity until specific value can increase the growth rate then increase in it causes the decrease in the growth rate because of the exiting of electrons from interactional zone via ponderomotive force. Also, effect of essential parameters such as external magnetic field, state of polarization and pulse length on the instability are investigated.

Keywords: Laser-plasma interactions, Modulational instability, Nonlinear Schrodinger equation

1 Introduction

Modulational instability (MI) is one of the fundamental phenomena in the nonlinear waves theory; the phenomenon that plays major role in different kinds of the nonlinear processes such as envelope solitons, envelope shocks, freak waves, etc. Pondermotive force originated from the electromagnetic (EM) wave stimulates low frequency perturbations of the electron density; then, they interact with the primary high frequency EM wave in which the amplitude of the pump wave becomes modulated, and the MI of the EM wave occurs. This phenomenon was predicted by Benjamin and Feir [1] for hydrodynamical waves and by Bespalov and Talanov [2] for EM waves in the nonlinear media with a cubic nonlinearity. The examples of MI from water wave hydrodynamics, electrodynamics, nonlinear optics, and convection theory can be found in Ref. [3]. The MI of a laser pulse in the cold plasma has been studied in several works [4]. Already, effect of temperature on the MI in quasi-neutral plasma has been investigated by Sepehri Javan [5]. In this work, we have studied the effect of plasma wake-field caused by the ponderomotive force on the MI.

2 Deriving Nonlinear Wave Equation

We consider the propagation of circularly polarized EM wave along the external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{e}}_z$ in the hot plasma. From Maxwell's equation, we can write wave equation as

$$\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = \frac{4\pi}{c}\mathbf{J}, \qquad (1)$$

where $\mathbf{A}$ is the vector potential, c is the speed of light, $\mathbf{J} = -(n_q + n_w)e\mathbf{v}_\perp$ is the current density of electrons of plasma, n_q is the density of electron in the quasi-neutral approximation, n_w is the density of electrons caused by wake-field, $\mathbf{v}_\perp$ is the transversal velocity of electron, and e is the magnitude of electron charge. Now, we write the relativistic fluid momentum equation for electrons

$$\frac{\partial\mathbf{p}}{\partial t} + \frac{1}{\gamma_e m_0}(\mathbf{p}.\nabla)\mathbf{p} = \frac{e}{c}\frac{\partial\mathbf{A}}{\partial t} + e\nabla(\varphi_a + \varphi_w) - \frac{e}{\gamma_e m_0 c}\mathbf{p}\times\nabla\times\mathbf{A} - \frac{\omega_c}{\gamma_e}\mathbf{p}\times\hat{\mathbf{e}}_z - k_B T_e\nabla\ln n_q, \qquad (2)$$

where $\mathbf{p} = \mathbf{p}_\perp + \mathbf{p}_z$ is the momentum of electron, $\gamma_e = \sqrt{1 + p^2/m_0^2c^2}$ is the relativistic Lorentz factor of electron, m_0 is the electron rest mass, φ_a and φ_w are ambipolar and wake-field scalar potentials, respectively, $\omega_c = eB_0/m_0c$ is the electron cyclotron frequency, k_B is the Boltzmann constant and T_e is the temperature of electrons. We consider the vector potential of laser wave as following

$$\mathbf{A} = \frac{1}{2}\tilde{A}(\hat{e}_x + i\sigma\hat{e}_y)\exp(-i\omega_0 t + ik_0 z) + c.c., \qquad (3)$$

where ω_0, k_0 are the frequency and wave number, $\sigma = +1,-1$ denotes the right- and left-hand circularly polarized wave, and $\tilde{A}(z,t)$ is the slowly varying amplitude. Inserting “Equation (3)” into “Equation (2)”, and assuming $v_z << v_{ph}$, we can find that “Equation (2)” is satisfied by [5]

$$\bar{\mathbf{p}}_\perp = \frac{\bar{\mathbf{A}}}{1-\sigma\alpha/\gamma_e}, \qquad (4)$$

$$\{\nabla[\Phi_a - \beta_e(\gamma_e + \frac{\sigma\alpha}{2\gamma_e^2}) - \ln(\frac{n_q}{n_0})]\}.\hat{\mathbf{e}}_z = 0, \qquad (5)$$

Where $\bar{\mathbf{p}}_\perp = \mathbf{p}_\perp/m_0c$, $\bar{\mathbf{A}} = e\mathbf{A}/m_0c^2$, $\Phi_a = e\varphi_a/k_B T_e$, $\alpha = \omega_c/\omega_0$ and n_0 is unperturbed density. Integrating “Equation (5)”, we can write

$$n_q = n_0\exp[-\kappa(\gamma_e - 1 - \sigma\alpha|\bar{\mathbf{p}}_\perp|^2/2\gamma_e^2)], \qquad (6)$$

Where $\kappa = \beta_e/1+\delta^{-1}$, $\delta = T_e/T_i$, $\beta_e = c^2/V_{Te}^2$ and $V_{Te}^2 = k_B T_e/m_0$. Neglecting nonlinear terms of p_z and taking into account wake-field potential φ_w, we can write

$$\frac{1}{c}\frac{\partial\bar{\mathbf{p}}_z}{\partial t} = \nabla\Phi_w - \frac{1}{2\gamma_e(1-\sigma\alpha/\gamma_e)}\nabla|\bar{\mathbf{A}}|^2, \qquad (7)$$

where $\bar{\mathbf{p}}_z = \mathbf{p}_z/m_0c$, and $\Phi_w = e\varphi_w/m_0c^2$. Now, we write the continuity equation as below

$$\frac{\partial}{\partial t}(n_q + n_w) + \nabla.[(n_q + n_w)\mathbf{v}_e] = 0, \qquad (8)$$

Sentences such as $n_w\nabla.\mathbf{v}_e$ and $\mathbf{v}_e.\nabla n_w$ are weak and we neglect them. Also, we neglect from spatial-temporal variations of n_q. For weakly relativistic laser intensity, we can write $\gamma_e = 1 + |\bar{\mathbf{p}}|^2/2$. Finally, continuity equation will be as

$$\frac{1}{c}\frac{\partial n_w}{\partial t} + n_q\nabla.\bar{\mathbf{p}}_z = 0. \qquad (9)$$

Now, we write Poisson's equation as below

$$\nabla^2\Phi_w = k_p^2 n_w / n_0 \,, \tag{10}$$

where $k_p = \omega_p / c = \sqrt{4\pi n_0 e^2 / m_0 c^2}$. Combining "Equations (7), (9), and (10)", yields to

$$\left(\frac{\partial^2}{c^2\partial t^2} + k_p^2\right)\frac{n_{\mathrm{w}}}{n_0} = \frac{1}{2\gamma_e(1-\sigma\alpha/\gamma_e)}\nabla^2 |\overline{\mathbf{A}}|^2 . \tag{11}$$

The solution to "Equation (11)" is

$$\frac{n_{\mathrm{w}}}{n_0} \approx \frac{c^2\int_0^t \sin[\omega_p(t-t')]\nabla^2 |\overline{\mathbf{A}}|^2 \, dt'}{2\omega_p\gamma_e(1-\sigma\alpha/\gamma_e)} \,, \tag{12}$$

Let us consider a normalized intensity profile as

$$\left|\overline{\mathbf{A}}\right|^2 = \left|\tilde{A}\right|^2 = A_0^2 \sin^2(\pi\xi / L)\exp(-2r^2/r_s^2) \,, \tag{13}$$

where L, r, r_s are pulse length, radial coordinate in cylindrical system, and spot size, respectively $\xi = z - V_g t$. Thus, density of wake-field will be

$$n_{\mathrm{w}} = \frac{2\pi^2c^2A_0^2n_0}{L^2[(2\pi V_g/L)^2-\omega_p^2]}\frac{\exp(-2r^2/r_s^2)}{(\gamma_e-\sigma\alpha)}\times \left\{1+\frac{8}{r_s^2k_p^2}\left(1-\frac{2r^2}{r_s^2}\right)\right\}\sin\left[\omega_p\left(\frac{\xi}{V_g}+\frac{L}{2V_g}\right)\right]\sin\left(\frac{\omega_p L}{2V_g}\right) \,, \tag{14}$$

From "Equation (4)", we can obtain

$$\mathbf{v}_\perp = (\gamma_e - \sigma\alpha)^{-1} e\mathbf{A}/(m_0 c) \,, \tag{15}$$

Taking "Equations (6), (14), and (15)" into consideration, for the weakly relativistic laser intensity, we can derive the nonlinear current density as the following

$$\frac{4\pi}{c}\mathbf{J} = -\frac{\omega_p^2}{c^2}\mathbf{A}P\{\exp\left(-\frac{\kappa}{2}\frac{|\overline{\mathbf{A}}|^2}{(1-\sigma\alpha)}\right) + \frac{2\pi^2c^2|\overline{\mathbf{A}}|^2}{L^2[\omega_P^2-(2\pi V_g/L)^2]}P\exp(-2r^2/r_s^2) \left[1+\frac{8}{r_s^2k_p^2}\left(1-\frac{2r^2}{r_s^2}\right)\right]\sin\left[\omega_p\left(\frac{\xi}{V_g}+\frac{L}{2V_g}\right)\right]\sin\left(\frac{\omega_p L}{2V_g}\right)\} \,, \tag{16}$$

where $P = (1-\sigma\alpha)^{-1}(1-|\overline{\mathbf{A}}|^2(1-\sigma\alpha)^{-3}/2)$. Substituting "Equation (13)" into "Equation (1)", integrating it with respect to *r*, saving only second order of $|\overline{\mathbf{A}}|$ and exerting the condition of slowly varying amplitude, we finally obtain

$$\left(i\overline{\omega}_0 + \frac{\pi\overline{V}_g}{\overline{L}}\cot\left(\frac{\pi\zeta}{\overline{L}}\right)\right)\frac{\partial a}{\partial\tau} + \frac{\pi}{\overline{L}}\cot\left(\frac{\pi\zeta}{\overline{L}}\right)(1-\overline{V}_g^2)\frac{\partial a}{\partial\zeta} + \frac{1}{2}\frac{\partial^2 a}{\partial\zeta^2} + \overline{D}_{NL}a = 0 \,, \tag{17}$$

where $\tau = \omega_p t$, $\zeta = \omega_p\xi / c$, $\overline{\omega}_0 = \omega_0/\omega_p$, $\overline{L} = \omega_p L / c$, $\overline{V}_g = V_g / c$, $a = e\tilde{A}/m_0 c$,

$$\overline{D}_{NL} = \frac{5|a|^2\sin^2(\pi\zeta/\overline{L})}{32(1-\sigma\alpha)^4}\exp\left(-\frac{\kappa|a|^2\sin^2(\pi\zeta/\overline{L})}{2(1-\sigma\alpha)}\right) - \frac{\pi^2|a|^2}{\overline{L}^2[1-(2\pi\overline{V}_g/\overline{L})^2]}\frac{1}{(1-\sigma\alpha)^2}\left(\frac{1}{3}+\frac{8}{9\bar{k}_p^2\bar{r}_s^2}\right)\times \sin[(\zeta+\overline{L}/2)/\overline{V}_g]\sin(\overline{L}/2\overline{V}_g)$$

, and $\bar{r}_s = \omega_p r_s / c$, $\bar{k}_p = ck_p/\omega_p$.

3 Modulational Instability

To derive the dispersion relation for MI, we use the well-known method in which we suppose

$$a = (a_0 + a_1)\exp(i\Lambda\tau + i\eta\zeta) \,, \tag{18}$$

where a_0 is a real parameter, $a_0 >> |a_1|$, and

$$\eta = -\overline{\omega}_0(\overline{V}_g - 1/\overline{V}_g) \pm \sqrt{\overline{\omega}_0^2(\overline{V}_g - 1/\overline{V}_g)^2 + 2\theta} \,,$$

$$\Lambda = (\overline{V}_g - 1/\overline{V}_g)\eta \,, \tag{19}$$

and also $\theta = \overline{D}_{NL(a=a_0)}$. Using "Equation (18)" in "Equation (17)" and linearizing it we can obtain

$$\left(i\overline{\omega}_0 + \frac{\pi\overline{V}_g}{\overline{L}}\cot\left(\frac{\pi\zeta}{\overline{L}}\right)\right)\frac{\partial a_1}{\partial\tau} + \left(\frac{\pi}{\overline{L}}(1-\overline{V}_g^2)\cot\left(\frac{\pi\zeta}{\overline{L}}\right) + i\eta\right)\frac{\partial a_1}{\partial\zeta} + \frac{1}{2}\frac{\partial^2 a_1}{\partial\zeta^2} + \theta(a_1 + a_1^*) = 0. \tag{20}$$

We assume

$$a_1 = X + iY \,, \tag{21}$$

where $X = \tilde{X}e^{-i\Omega\tau + iK\zeta}$, $Y = \tilde{Y}e^{-i\Omega\tau + iK\zeta}$. Using "Equation (21) in "Equation (20)" leads to the following dispersion relation

$$\left[\overline{\omega}_0^2 + \frac{\pi^2\overline{V}_g^2}{\overline{L}^2}\cot^2\left(\frac{\pi\zeta}{\overline{L}}\right)\right]\Omega^2 - \left[2K\eta\overline{\omega}_0 - \frac{2i\theta\pi\overline{V}_g}{\overline{L}}\cot\left(\frac{\pi\zeta}{\overline{L}}\right) + \frac{2\pi^2\overline{V}_gK}{\overline{L}^2}(1-\overline{V}_g^2)\cot^2\left(\frac{\pi\zeta}{\overline{L}}\right) + \frac{i\pi\overline{V}_gK^2}{\overline{L}}\cot\left(\frac{\pi\zeta}{\overline{L}}\right)\right]\Omega + K^2\eta^2 + \frac{\pi^2K^2}{\overline{L}^2}(1-\overline{V}_g^2)^2\cot^2\left(\frac{\pi\zeta}{\overline{L}}\right) - \frac{K^4}{4} + K^2\theta + \frac{i\pi K^3}{\overline{L}}(1-\overline{V}_g^2)\cot\left(\frac{\pi\zeta}{\overline{L}}\right) - \frac{2i\pi\theta K}{\overline{L}}(1-\overline{V}_g^2)\cot\left(\frac{\pi\zeta}{\overline{L}}\right) = 0 \,, \tag{22}$$

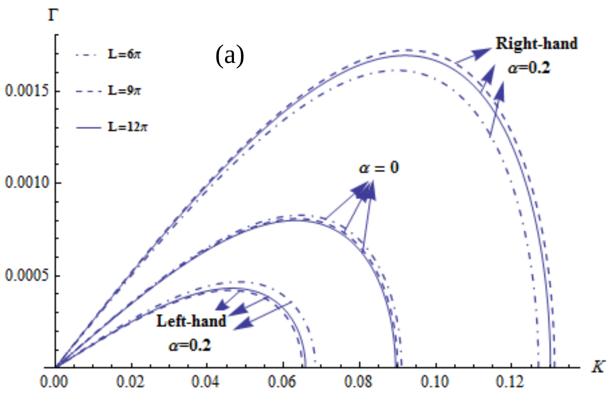
The positive imaginary part of frequency in this dispersion relation is the growth rate of MI.

4 Numerical Discussions

We have supposed Nd:YaG laser with frequency $\omega_0 = 1.88\times10^{15}\,\mathrm{s}^{-1}$, $a_0 = 0.1$ and $r_s = 15\mu m$. Figure 1-a shows variation of growth rate with respect to K for three different values of the pulse length when $\overline{\omega}_0 = 2.5$, $\alpha = 0.2$ and $a_0 = 0.15$. It is observed that the growth rate increases with exerting external magnetic field for the right-hand polarization. Inversely, for the left-hand polarization, growth rate decreases by using magnetic field. Also we can see that the growth rate with increasing pulse length acquires different values. In figure 1-b we have plotted growth rate as a function of K for three different values of the laser pulse intensity with $\overline{\omega}_0 = 10$, $\alpha = 0.2$ and $\overline{L} = 20\pi$. In this case growth rate increases with the increase of the laser pulse intensity until specific value, then increase of the intensity causes decrease of the growth rate.

5 Conclusions

We have investigated the MI of short laser pulse in hot magnetized plasma. Effect of external magnetic field, state of polarization, pulse length, and laser pulse intensity on the instability has been studied. It is observed that existence of magnetic field enhances the growth rate of the instability for the right-hand polarization. Inversely, for the left-hand polarization, magnetization of plasma causes the decrease of growth rate. The growth rate increases with the increases of the laser pulse intensity until specific value, then because of the exiting of electrons from interactional zone via ponderomotive force, increase of the intensity causes decrease of the growth rate.



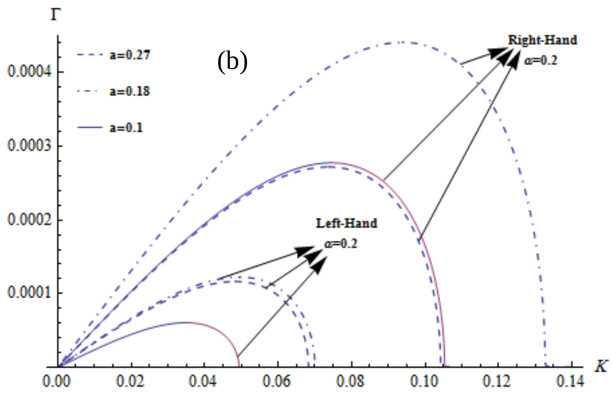


Figure 1: growth rate Γ as a function of K for (a) three different pulse length and $\sigma = \pm 1, 0$ (b) three different laser pulse intensity and $\sigma = \pm 1$.

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