



## طراحی یک موجبر نوری بدون پراکندگی بر پایه عایق‌های توپولوژیک فوتونی

محمد رضا مهدی‌پور، وحید احمدی، رضا پور صالحی

تهران - بزرگراه جلال آل احمد - دانشگاه تربیت مدرس - دانشکده مهندسی برق و الکترونیک

چکیده- عایق‌های توپولوژیک فوتونی سیستم‌هایی هستند که در آن‌ها مدهای محافظت شده لبه‌ای با شکستن تقارن وارونی زمانی پدیدار می‌شوند. این مدها در گاف انرژی غیر بدینه بوجود می‌آیند و امواج الکترومغناطیسی بدون پراکندگی امکان انتشار پیدا می‌کنند. در این مقاله یک موجبر نوری یک طرفه با استفاده از مدهای توپولوژیک در بازه فرکانسی ۶ تا ۸ گیگاهرتز برو سی می‌شود. تقارن وارونی زمانی با استفاده از یک میدان مغناطیسی خارجی در جهت  $z$  شکسته می‌شود. در این مقاله نشان داده می‌شود که این موجبر نور را تنها در یک جهت انتشار می‌دهد که قابل کنترل توسط میدان مغناطیسی اعمال شده می‌باشد. این موجبر امکان هدایت نور را به صورت یک طرفه فراهم می‌کند به صورتی که ۱۰۰ درصد نور به سمت خروجی هدایت می‌شود و هیچ پراکندگی در آن وجود ندارد.

کلید واژه- عایق توپولوژیک فوتونی، بلور فوتونی، انتشار یک طرفه، مدهای لبه‌ای

### Design of a Backscatter-Free Waveguide Based on Photonic Topological Insulators

Mohammadreza Mehdipour, Vahid Ahmadi\*, Reza Poursalehi

Tarbiat Modares University, Jalal-E-Al Ahmad St. , Tehran, Iran

**Abstract-** Photonic Topological Insulators are systems where the broken time reversal symmetry gives rise to protected edge modes that support backscatter-free and one-way propagation of electromagnetic waves by opening non-trivial bandgaps. In this study we investigate a one-way topologically protected waveguide in the frequency range of  $f=6.0$  to  $8.0$  GHz. The time reversal symmetry is broken by an applied magnetic field in the  $z$  direction. We show that the waveguide propagates the light in only one direction that can be controlled by the applied magnetic field and no backscattering is present in the waveguide which results in a near 100% transmission of light to the output.

Keywords: Photonic Topological Insulators, Photonic Crystals, One way Propagation, Edge Modes



## 1. Introduction

Topological photonics has been a promising field of research, where edge state modes exhibit in the system and provide one-way propagation of electromagnetic fields which are robust against impurities and defects and no backscattering is allowed[1-5]. Photonic band gaps appearing in photonic crystals (PCs) has inspired many applications in integrated optics. Namely, PCs made by magneto-optical materials are used in fabricating non-reciprocal optical circuits[6, 7]. Recently, backscatter-free gyromagnetic photonic crystal waveguides have drawn much attention due to their potential applications in circulators and switches. The modes, defined by edge states that have group velocities directed in only one direction are confined to the edge of the magneto-optical photonic crystal and can be controlled by an applied magnetic field. Hence, no backscattering exists in such systems due to the lack of back propagating modes. In order to achieve topological protected edge states time reversal symmetry is broken in the system by an applied magnetic field. In this paper we propose a waveguide and investigate the effect of the applied magnetic field on the topological edge states of the system.

## 2. Theory and Simulation

The basic structure is a 2D honeycomb photonic crystal with radius  $0.3a$ , where  $a$ , is the lattice constant (Fig. 1(a)). In Fig. 1(a) each circle represents a rod with relative permittivity of  $\epsilon = 15\epsilon_0$ . When a DC magnetic field is applied along the axis of the rods, the field would induce the magnetic anisotropy of the ferromagnetic rods. After the material is fully magnetized, the magnetic permeability tensor  $\mu$  is given by

$$\bar{\mu} = \begin{bmatrix} \mu_r & -i\mu_k & 0 \\ i\mu_k & \mu_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where

$$\mu_r = 1 + \frac{\omega_m \omega_0}{\omega_0^2 - \omega^2}, \quad (2)$$

$$\mu_k = \frac{\omega_m \omega}{\omega_0^2 - \omega^2}, \quad (3)$$

The precession frequency is

$$\omega_0 = \gamma H_0. \quad (4)$$

where  $\gamma = 2.8 \times 10^{-3} \text{ C/Kg}$  is the gyromagnetic ratio and  $H_0$  is the applied magnetic field.

The dispersion relation of the structure is calculated using the conventional band theory of photonic crystals. In order to have one-way edge states, one must break the time reversal symmetry of the magneto optical photonic crystal modes which directly affects the topological properties of the bands of the system. This effect is generally defined by the Chern number, previously studied in quantum Hall effect extensively. The Chern number of the n-th band is defined as

$$C_n = \frac{1}{2\pi i} \int_{BZ} d^2 k \left( \frac{\partial A_y^{nn}}{\partial k_x} - \frac{\partial A_y^{nn}}{\partial k_y} \right) \quad (5)$$

The integral in (5) is over the entire Brillouin zone and  $A^{nn}$  is the Berry phase defined as

$$A^{nn}(k) = \langle E_{nk} | \nabla_k | E_{nk} \rangle \quad (6)$$

which is performed over the unit cell. Here  $E_{nk}$  is the eigenvalue of the n-th band.

Each band has a Chern number which is shown to be always an integer, and if the band is time reversal symmetric the Chern number in Equation (5) is always equal to zero. It is also possible to adiabatically tune the Hamiltonian by changing the permeability tensor of the system. In this case, the

Chern number of the system can be tuned by the applied magnetic field. In fact, this change happens where the band is degenerate and by applying the magnetic field the degeneracy is lifted and discrete degenerate points show up, a bandgap opens and the Chern number of the band changes by  $\Delta C$ . Thus, we look for a time reversal symmetric band structure with a pair of photonic bands degenerate at a specific wave vector.

The band structure for the applied magnetic field  $H=0.1$  T around  $k=0$  is depicted in Figure 1(b) where a band gap is shaped by the broken time reversal symmetry. The solid lines represent the eigenvalues of the photon energy inside the photonic crystal structure where a gap is opened in frequency range  $f=7.0$  to  $f=7.3$  GHz at  $k=0$ . However the dashed lines inside the gap represent the edge modes of the photons that have different Chern number of  $\Delta C=1$ . Hence giving rise to the topologically protected states that can propagate in one direction only, and are immune to backscattering.

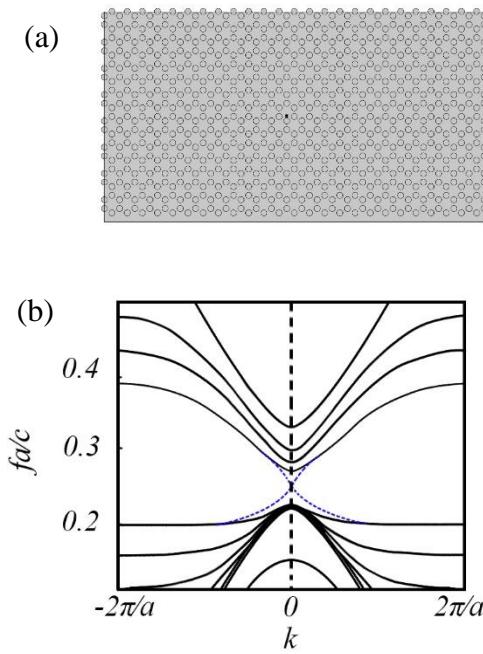


Fig. 1 : (a) The honeycomb structure of ferrite rods is air with lattice constant  $a$ . (b) Dispersion relation of the honeycomb structure. Time reversal symmetry is broken in the band by applying a magnetic field, leading to one-way edge states inside the bandgap.

### 3. Result and Discussion

#### 3.1. Designing a waveguide

In order to design a waveguide, we consider the structure depicted in Figure 1(a) adjacent with another photonic crystal with the same geometry but the rods have a permeability tensor with mirrored off-diagonal elements with that of Equation (1).

Figure 2 shows the structure of the waveguide where the blue (top) domain is the same as Figure 1(a) and the red (bottom) domain has mirrored off-diagonal elements with respect to the blue (top) domain. The space created between the two domains acts as a waveguide for the frequency range inside the bandgap of the top and bottom photonic crystals and only edge modes will be allowed to propagate inside the waveguide, which are topologically protected and can only propagate in one direction only.

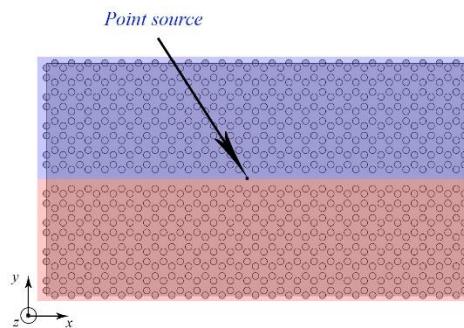


Fig. 2 : Two domains with mirrored off-diagonal elements in their permeability tensor can form a one-directional waveguide

To confirm the one-way topologically protected edge modes we introduce a point source in the middle of the waveguide formed by two domains (Figure 2). The top domain only supports modes that can propagate around it clockwise and the bottom domain only supports modes that propagate around it in a counter clockwise direction. As shown in Figure 3(a) it can be seen that frequencies with different Chern number only propagate to the left and no propagation of light is evident as there are no modes to support propagation to the right. Therefore, it is confirmed that a one-way

propagating waveguide can be formed by closing in the two domains with mirrored magnetic permeability tensors. By changing the direction of the applied magnetic field to  $-z$ , it can be seen that the propagating modes also change their direction (Figure 3(a) inset). Also, we calculate the energy received at the left and right port of the waveguide when a  $H=0.1$  (T) magnetic field in the  $z$  direction is applied to the system (Figure 3(b)). It can be clearly seen that at  $f=7.25$  GHz a near 100% transmission of light is received at the left port where at the same frequency the energy received at the right port drops to zero. This can confirm that this is a novel design for one-way propagating waveguide where there are no back scatterings allowed in the desired frequency range. This unique feature can be promising in the on/off switches since the light output can be controlled by the direction of the applied magnetic field.

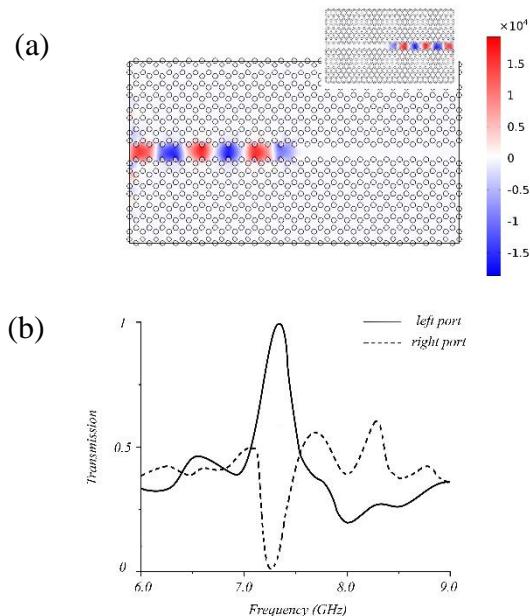


Fig. 3 : (a) The electric field profile at  $f=7.25$  GHz shows that propagation of the electromagnetic field is only allowed to the left in the waveguide. The inset shows the direction of the propagating light can change by the direction of the applied magnetic field. (b) The transmission spectrum of the waveguide shows that at  $f=7.25$  GHz all the energy is propagated to the left port and zero energy is received at the right port.

#### 4. Conclusion

We have proposed a waveguide based on photonic topological insulators by putting two different domains of honeycomb photonic crystals with mirrored magnetic permeability tensors near each other. By breaking the time reversal symmetry in the system, protected edge modes appear that only support propagation of light in one direction in the waveguide. It can be seen that for an applied magnetic field of  $H=0.1$  T protected edge modes allow propagation of light in the frequency range of  $f=7.04$  GHz to  $f=7.32$  GHz to one side only with no backscattering in the system. In this frequency range all the energy is delivered to the left port and zero energy reaches the right port which confirms the one-way edge modes of the system.

By changing the direction of the applied magnetic field the direction of the propagating modes can be controlled which can be used as an on/off switch either. In general the ability to control the light perfectly can give rise to infinite applications in future optical integrated circuits.

#### 5. References

- [1] Mele, C.L.K.a.E.J., *Z2 topological order and quantum spin hall effect*. *Physical Review Letter*, 2005. 95(146802).
- [2] N. H. Lindner, G.R.a.V.G., *Floquet Topological insulator in semiconductor quantum wells*. *Nat Phys*, 2011. 7.
- [3] T. Ma, A.B.K., S. H. Mousavi, and G. Shvets, *Guiding electromagnetic waves around sharp corners: Topologically protected photonic transport in metawaveguides*. *Physical Review Letters*, 2015. 114(12).
- [4] Zhang, X.L.Q.a.S.C., *Topological insulators and superconductors*. *Review Modern Physics*, 2011. 83.
- [5] S. A. Skirlo, L.L., and M. Soljacic., *Multimode one-way waveguides of large chern numbers*. *Physical review letters*, 2014. 113(11).
- [6] Z. Wang, a.S.F.F., *Opt. Lett.*, 2005. 30(1989).
- [7] P. Wang, L.L., and K. Bertoldi., *Topological phononic crystals with one-way elastic edge waves*. *Physical review letters*, 2015. 115(10).