



## خودکانونگی پالس لیزر منتشرشونده در یک محیط حجیم مغناطیده از نانوذرات گرافیت

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چکیده - در این مقاله خودکانونگی پالس لیزر پرشدت منتشر شونده در یک محیط حجیم مغناطیده شامل نانوذرات گرافیت مطالعه شده است. با استفاده از روش اختلالی، معادله‌ی موج توصیف کننده‌ی اندرکنش غیرخطی میدان‌های لیزر پرشدت با نانوذرات مغناطیده‌ی گرافیت حاصل شده است. تحولات لکه‌ی لیزری برای قطبش دایروی با ساختار گوسی به دست آمده است. توجه ویژه‌ای به نقش میدان مغناطیسی خارجی بر خودکانونگی شده است.

کلید واژه- لیزر، شبکه‌ی نانوذرات، معادله‌ی موج غیرخطی، خودکانونگی، لکه‌ی لیزری

## Self-Focusing of an Intense Laser Pulse Propagating in a Magnetized Bulk Medium of Graphite Nanoparticles

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Abstract- In this paper self-focusing of an intense laser pulse propagating through a magnetized bulk medium containing graphite nanoparticles is studied. Using a perturbative method, wave equation describing nonlinear interaction of laser fields with graphite magnetized nanoparticles is derived. Evolution of laser spot size for the circular polarization with Gaussian profile is considered. An especial attention is paid on the role of external magnetic field in the self-focusing.

Keywords: laser, nanoparticles lattice, nonlinear wave equation, self-focusing, spot size

# Self-Focusing of an Intense Laser Pulse Propagating in a Magnetized Bulk Medium of Graphite Nanoparticles

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## 1 Introduction

Nonlinear interaction of laser with one-dimensional periodically nanostructured metals can lead to the localization or focusing of light in a region with characteristic dimensions smaller than the incident wavelength. This phenomenon called nano-focusing. Such a nano-focusing property is observed theoretically and experimentally in the interaction of laser with determinate number or chain of nanoparticles [1,2]. Moreover, localization of intense Electromagnetic Wave (EMW) can occur macroscopically during interaction of laser with a bulk medium consisting nanoparticles via so-called well-known phenomenon Self-Focusing (SF) [3] that appears via the change of the medium refractive index exposed to the intense EM radiation. SF of unmagnetized bulk medium including metallic NPs has been studied theoretically by Sepehri Javan [4]. In this paper we have studied effect of external magnetic field on the SF of laser in the interaction with a bulk medium consisting of spherical graphite nanoparticles.

## 2 Nonlinear wave equation

We consider the propagation of an intense EMW through a magnetized bulk medium of graphite nanoparticles with average radius  $r_c$  and average separation  $d$ . We suggest that the external magnetic field is along the  $z$  axis  $\mathbf{B}_0 = B_0 \hat{\mathbf{e}}_z$ . Electric and magnetic fields of the laser beam are as bellow

$$\mathbf{E}_L = \mathbf{E}^{(1)} = \frac{\hat{E}}{2} (\hat{\mathbf{e}}_x + i\sigma \hat{\mathbf{e}}_y) e^{-i(\omega t - kz)} + c.c., \quad (1)$$

$$\mathbf{B}_L = \mathbf{B}^{(1)} = -\frac{i\sigma k c \hat{E}}{2\omega} (\hat{\mathbf{e}}_x + i\sigma \hat{\mathbf{e}}_y) e^{-i(\omega t - kz)} + c.c., \quad (2)$$

where,  $\hat{E}, \omega, kc$  are the slowly varying amplitude, frequency and wave number of the laser, speed of light and  $\sigma = +1, -1$ , denotes the right- and left-hand circularly polarized waves, respectively. Equation describing the relativistic interaction of laser fields with electronic cloud of each nanoparticle can be written as

$$d(\gamma \mathbf{v})/dt + \Gamma \mathbf{v} + \omega_p^2 \mathbf{r}/3 = -(e/m) \{ \mathbf{E}(0) + (\mathbf{r} \cdot \nabla) \mathbf{E}(0) + \mathbf{v} \times [(\mathbf{B}(0) + \mathbf{B}_0) + (\mathbf{r} \cdot \nabla) \mathbf{B}(0)] / c \} \quad (3)$$

where  $\gamma$ ,  $\mathbf{v}$ ,  $\mathbf{r}$ ,  $\omega_p = (4\pi n_0 e^2 / m)^{1/2}$ ,  $\Gamma$ ,  $e$  and  $m$  are the relativistic Lorentz factor, velocity, displacement of the electron cloud from the equilibrium state, electron plasma frequency, damping factor related to electron scattering, magnitude of electron charge and electron rest mass, respectively,  $\mathbf{E}(0)$  and  $\mathbf{B}(0)$  represent the electromagnetic fields in the center of the particle. First order momentum equations is

$$d\mathbf{v}^{(1)} / dt + \Gamma \mathbf{v}^{(1)} + \omega_p^2 \mathbf{r}^{(1)} / 3 = -(e/m) \{ \mathbf{E}^{(1)}(0) + (\mathbf{v}^{(1)} \times \mathbf{B}_0) / c \} \quad (4)$$

where superscripts (1) refer to the first order perturbed parameters. The solution of Equation (4) is

$$\mathbf{v}^{(1)} = -\frac{i}{2} \frac{ac\omega^2 e^{-i(\omega t - kz)} (\hat{\mathbf{e}}_x + i\sigma \hat{\mathbf{e}}_y)}{(\omega^2 + i\Gamma\omega - \omega_p^2/3 - \sigma\omega\omega_c)} + c.c., \quad (5)$$

where  $a = e\hat{E}/mc\omega$  is the normalized laser amplitude. We know that circular EMW cannot cause the second order displacement of electrons. The third-order equation of the electron cloud movement is

$$d[\mathbf{v}^{(3)} + \gamma^{(2)} \mathbf{v}^{(1)}]/dt + \Gamma \mathbf{v}^{(3)} + \omega_p^2 \mathbf{r}^{(3)}/3 = -e(\mathbf{v}^{(3)} \times \mathbf{B}_0)/mc \quad (6)$$

From Equation (6) we can obtain

$$\mathbf{v}^{(3)} = ia^3 c \omega e^{-i(\omega t - kz)} (\hat{\mathbf{e}}_x + i \hat{\mathbf{e}}_y) \times \frac{[\omega^2 + i\Gamma\omega - \omega_p^2/3 - \sigma\omega\omega_c]^{-2}}{4[(\omega^2 - \omega_p^2/3 - \sigma\omega\omega_c)^2 + \Gamma^2\omega^2]} + c.c. \quad (7)$$

Graphite consists of atoms regularly located on planes (basal planes) which are equally separated from each other. Quality of EMW propagation is different in the case of parallel or perpendicular orientation of electric field with respect to the graphite basal plane. We will use indices  $\perp$  and  $\parallel$  for the configurations  $E \perp \hat{\mathbf{n}}$  and  $E \parallel \hat{\mathbf{n}}$ , respectively, where  $\hat{\mathbf{n}}$  is a unit vector normal to the basal plane. Now we consider a bulk medium including equal amounts of two different sorts of graphite nanoparticles, i. e.  $\perp$  and  $\parallel$  orientations of basal planes. We suppose that statistically both kinds of nanoparticles have same average radius and separation. For such a medium we can write the following wave equation

$$(\nabla^2 - \frac{\epsilon_{eff}}{c^2} \frac{\partial^2}{\partial t^2}) \mathbf{E} = \frac{4\pi}{c^2} \frac{\partial \mathbf{J}}{\partial t}, \quad (8)$$

where  $\mathbf{J} = -\sum_{s=\parallel, \perp} (4\pi e l_s/3) (n_{0s}) (\mathbf{v}^{(1)} + \mathbf{v}^{(3)})$  is the total macroscopic current density, index  $s$  denotes sort of orientation of basal plane of nanoparticle,  $n_0$  is the electron density of the electron cloud,  $l_s = (r_{cs}/d_s)^3$  and  $\epsilon_{eff}$  is the effective permittivity related to the bound electrons of medium. Using Equation (1) and multiplying both sides of Equation (11) by  $e/mc\omega$  yields

$$\left[ \nabla^2 - \frac{\epsilon_{eff}}{c^2} \frac{\partial^2}{\partial t^2} \right] a e^{-i\omega t + ikz} = \left[ \frac{4\pi}{3c^2} H \right] a e^{-i\omega t + ikz} \quad (9)$$

where  $\omega_c = eB_0/mc$  is electron cyclotron frequency,

$$H = \sum_{s=\parallel, \perp} \left( \frac{\omega^2 \omega_p^2 l_s}{\omega^2 + i\Gamma_s \omega - \omega_{p,s}^2/3 - \sigma\omega\omega_c} - \omega_{p,s}^2 N_s |a|^2 \right)$$

and  $N_s = \frac{\omega^8 l_s [\omega^2 + i\Gamma_s \omega - \omega_{p,s}^2/3 - \sigma\omega\omega_c]^{-2}}{2[(\omega^2 - \omega_{p,s}^2/3 - \sigma\omega\omega_c)^2 + \Gamma_s^2 \omega^2]}$ . In

the absence of interaction, when  $a = a_0$  is a

constant, Equation (9) leads to the following nonlinear dispersion relation

$$D_{NL} = k^2 - \omega^2 \epsilon_{eff}/c^2 + (4\pi/3c^2) H = 0. \quad (10)$$

### 3 Envelope evolution

To study the problem of SF we use a method like the well-known source dependent expansion (SDE) method [5]. First, we introduce dimensionless electric field of laser as following

$$a(r, \theta, z, t) = a(r, \theta, z) e^{i(kz - \omega t)} / 2 + c.c., \quad (11)$$

Substituting Equation (11) in Equation (9) results

$$\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + 2i \frac{\omega}{c} \frac{\partial}{\partial z} \right] a(r, \theta, z) = S(r, \theta, z) \quad (12)$$

where the source term of the right hand side is

$$S(r, \theta, z) = \frac{\omega^2}{c^2} [1 - n^2(r, \theta, z, a)] a(r, \theta, z) \quad (13)$$

and nonlinear complex refractive index can be achieved from equation (10)

$$n^2 = \epsilon_{eff} - 4\pi H/3\omega^2. \quad (14)$$

By expansion of the amplitude  $a(r, \theta, z)$  as a series of associated Laguerre-Gaussian source-dependent modes [5], we obtain Equation (12) as

$$\frac{d^2 r_s}{dz^2} + 2C_2 \frac{dr_s}{dz} + C_2^2 r_s - \frac{C_3}{r_s^3} - \frac{C_4}{r_s} = 0, \quad (15)$$

where

$$C_1 = (F_{1,0}/a_{0,0})_{\text{Re}}, \quad C_2 = (F_{1,0}/a_{0,0})_{\text{Im}},$$

$C_3 = (2c/\omega)^2$ ,  $C_4 = (4cC_1/\omega)$  and indices Re and Im refer to the real and imaginary parts of any quantity.  $F_{1,0}$  and  $a_{0,0}$  have been introduced in Ref. [5] and for brevity we don't bring them here.

### 4 Numerical discussions

A numerical analysis has been carried out in order to solve equation (15) for finding spatial evolutions of laser spot size. It is well-known that effect of external magnetic field on the nonlinear property of circularly polarized left-hand laser beam is inverse [4,6] and increase in the magnetic field leads to decrease in the nonlinearity of medium and consequently to decrease in the self-focusing quality of laser, therefore we consider only the right-hand polarization. Figure (1) shows variations of normalized laser spot size  $r_s/r_0$  with respect to normalized propagation distance

$z/z_R$ , where  $z_R = kr_0^2/2$  is Rayleigh length, for different values of external magnetic field. It is worth mentioning that the laser frequency is chosen near the main effective resonance frequency related to the perpendicular basal plane, i. e.  $\omega_0 \approx 1.2\omega_{p\perp}/\sqrt{3} \approx 9.99 \times 10^{14} s^{-1}$ , where nonlinearity of medium resonantly increases. In this frequency area, Cobalt-MgF2 lasers are available. In Figure (1), for all cases, spot size decreases with respect to propagation length, reaches a minimum, oscillates getting damped around a value above the first minimum and finally relatively becomes constant. Decrease of spot size is equivalent to beam focusing and its being constant means that diffraction makes balance with convergence caused by nonlinearity and beam propagates without any considerable change in its shape. Such situation is called self-trapping or self-guiding regime [7]. By exerting external magnetic field, self-focusing property improves. One can see that gradually increase in the magnetic field results in decrease in the main minimum of spot size and shift of its place to smaller values as well. It means that applying magnetic field leads to more convergence and its earlier appearance that in turn means better focusing. Also, for magnetized cases, self-guiding regime occurs earlier. In Figure (2), we set laser frequency slightly less than resonance frequency ( $\omega_0 = 0.9\omega_{p\perp}/\sqrt{3} \approx 7.49 \times 10^{14} s^{-1}$ ). First, at low values of propagation length, laser beam focuses for both cases and then it starts diverging. We can see that application of external magnetic field causes earlier focusing of laser beam. We suppose that the morphological situation of medium for two different sorts of graphite nanoparticles is the same, i. e.  $l_{\parallel} = l_{\perp} = l$  and  $r_{c\parallel} = r_{c\perp} = r_c$ . For all cases we set  $r_0 = 15 \mu m$ ,  $I_L = 10^{15} Wcm^{-2}$ ,  $l = (1/6)^3$ . Using theoretical results of [8], we have following parameters for parallel and perpendicular cases of basal planes of graphite nanoparticles:  $\omega_{p\parallel} \approx 1.5 \times 10^{14} s^{-1}$ ,  $\omega_{p\perp} \approx 1.4 \times 10^{15} s^{-1}$ ,  $\Gamma_{\parallel} \approx 10^{14} s^{-1}$  and  $\Gamma_{\perp} \approx 10^{15} s^{-1}$ .

## 5 Conclusion

We investigated SF of an intense laser pulse in a magnetized bulk medium of graphite nanoparticles. Spot size evolution of right-handed circularly-polarized laser is studied. Effect of

external magnetic field on the SF is considered. It is observed that increase in the magnetic field causes improvement of focusing property of medium.

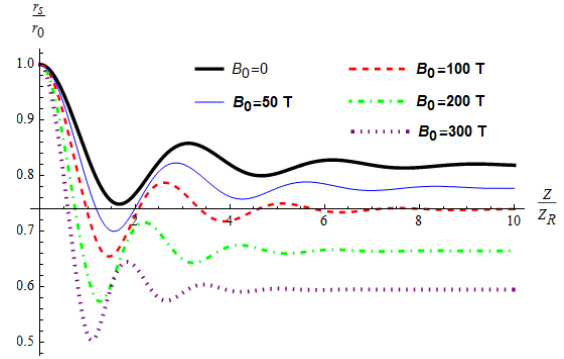


Figure 1: Variation of  $r_s/r_0$  with respect to  $z/z_R$  for five different values of external magnetic field.

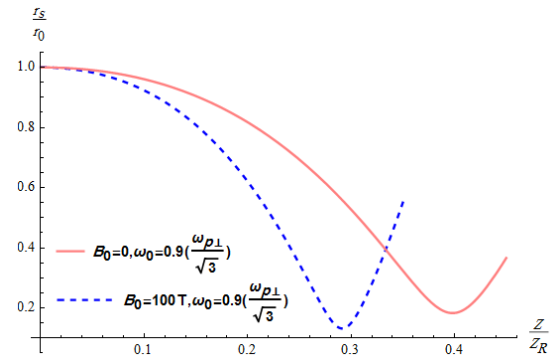


Figure 2: Variation of  $r_s/r_0$  with respect to  $z/z_R$  for unmagnetized and magnetized cases.

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