شکل‌گیری یک برهم‌نهی عمومی حالت‌های همدوس در حضور فوتون‌های مجازی

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چکیده – *دراین مقاله هامیلتونی برهم‌کنش اتم-فوتون، معروف به مدل جینز-کامینگز، بدون تقریب موج چرخان با استفاده از یک عملگر جابه‌جا شونده با آن قطری شده و بردارهای ویژه‌ی آن به‌صورت ترکیبی از برهم نهی حالت‌های همدوس نور و حالت‌های اتمی به‌دست آمده است. نهایتا نشان داده شده است که با انتخاب حالت اولیه به‌صورت ترکیب خطی بردارهای ویژه‌ی به‌دست آمده و محاسبه‌ی تحول زمانی آن، پس از اندازه-گیری حالت اتمی، حالت نور به‌صورت یک برهم‌نهی عمومی از حالت‌های همدوس به‌دست می‌آید. حالت‌های معروف گربه‌ی زوج و فرد و یورک-استولر با انتخاب حالت اولیه‌ی مناسب، محاسبه شده‌اند.*

کلید واژه- مدل جینزکامینگز، فوتون‌های مجازی، حالت‌های گربه، حالت یورک استولر.

Manifestation of a General Coherent State Superposition in the Presence of Virtual Photons

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Abstract- In this paper the atom-photon interaction Hamiltonian, known as the Jaynes-Cummings model, without the rotating wave approximation is diagonalized by an auxiliary operator that commutes with the Hamiltonian. The eigenstates obtained as a combination of the coherent light and the atomic states. It is shown that choosing the initial state as a linear combination of the eigenstates, and computing its time evolution and measuring the atomic states, sets the light state to a general superposition of the coherent states. The well-known Yurke-Stoler state and the even and odd cat states, was obtained as some examples of the method.

Keywords: Jaynes-Cummings model, Virtual photons, Cat States, Yurke-Stoler state.

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1 Introduction

Simultaneous existence of a cat in both death and live states introduced in Schrodinger's thought experiment [1] is impossible in real macroscopic world. Nevertheless it has been realized experimentally [2] through the interaction of atom with coherent state of light as the closest quantum mechanical state to the classical light. These experiments made it possible to study the boundaries of classical and quantum physics [2], and have been adopted to test the basic postulates of quantum mechanics.

Many theoretical methods have been proposed to construct quantum light superposition. The Yurke-Stoler state was obtained by the time evolution of coherent light under a time evolution operator based on a nonlinear Hamiltonian describing anharmonic oscillator [3]. Dispersive interaction in the Jaynes-Cummings model (JCM) when the atom and photon are largely detuned tends to the even and odd cat states [4].

In this paper, we obtain a general superposition of coherent states based on the exact solution of the JCM. The prominent point of our results is that despite previousely introduced methods it is free of any nonlinear effect and don't need any limitation on the interaction parameters. Nevertheless, the general superposition of coherent states converts to many known superposition states and cat states by suitable selection of initial states.

2 The JCM Solution

Consider a two level atom by the transition frequency ω_A interacting with a single mode of optical field of frequency ω_F for which the interaction intensity is given by λ. In a system of units for which $\hbar = 1$, this system describes by the Hamiltonian

$$\hat{H} = \frac{\omega_A}{2}\hat{\sigma}_z + \omega_F \hat{a}^\dagger \hat{a} + \lambda(\hat{a}^\dagger + \hat{a})(\hat{\sigma}_+ + \hat{\sigma}_-) \quad (1)$$

in which $\hat{\sigma}_z$ and $a^\dagger a$ are the atomic transient operator and the photon number operator, respectively. The field operator is given by $\hat{a}^\dagger + \hat{a}$ and $\hat{\sigma}_+ + \hat{\sigma}_- = \hat{\sigma}_x$ represents the atomic dipole operator. The product $(\hat{a}^\dagger + \hat{a})(\hat{\sigma}_+ + \hat{\sigma}_-)$ models the atomic dipole interaction that consists of the terms $\hat{a}^\dagger\hat{\sigma}_-$, $\hat{a}\hat{\sigma}_+$, $\hat{a}^\dagger\hat{\sigma}_+$ and $\hat{a}\hat{\sigma}_-$. First two terms describe the destruction of an exited atom by creating a photon and exiting an atom by absorbing a photon, respectively while the counter-rotating terms, $\hat{a}\hat{\sigma}_-$ and $\hat{a}^\dagger\hat{\sigma}_+$ describe excitation of an atom simultaneous with emission of a photon and destruction of an excited atom by absorbing a photon, respectively [5]. The photons included in counter-rotating terms are known as the virtual photons for which the statics and dynamics are studied in [6, 7].

We need to obtain the eigenstates of the JCM Hamiltonian given in the equation (1) as the substructures of this letter. Toward this end, we use the convention applied in [8], that is performing a $\pi/2$ Radian rotation of $\hat{H}$ around y axis, $\hat{R}(y, \pi/2)$, to obtain

$$\hat{H}_R = -\frac{\omega_A}{2}\hat{\sigma}_x + \omega_F \hat{a}^\dagger \hat{a} + \lambda(\hat{a}^\dagger + \hat{a})\hat{\sigma}_z \quad (2)$$

The rotated Hamiltonian $\hat{H}_R$, commutes with the parity operator, $\hat{\Pi} = \hat{\sigma}_x e^{i\pi\hat{N}}$, so they have the common eigenstates

$$|\psi_{R\pm}\rangle = \frac{1}{\sqrt{2}}[|e,\alpha\rangle,\ \pm|g,-\alpha\rangle] \quad (3)$$

with the eigenvalues $E_{\pm} = \mp\frac{\omega_A}{2}e^{-2\alpha^2} + \omega_F|\alpha|^2 + \lambda(\alpha+\alpha^*)$. To obtain the eigenstate of the Hamiltonian given by the equation (1), we just need to operate its eigenstates, $|\psi_{R\pm}\rangle$, with the operator, $\hat{R}(y,-\pi/2)$ to reverse the rotation. The results are

$$|\psi_{\pm}\rangle = \frac{1}{2}[|e,\alpha\rangle + |g,\alpha\rangle \pm (|g,-\alpha\rangle - |e,-\alpha\rangle)] \quad (4)$$

Factoring out the atomic states in the equation (4), and defining $|C_e\rangle = |\alpha\rangle + |-\alpha\rangle$ and $|C_o\rangle = |\alpha\rangle - |-\alpha\rangle$ reveals that $|\psi_+\rangle$ and $|\psi_-\rangle$ are combinations of the atomic states with the even and odd cat states, as follows

$$|\psi_+\rangle = \frac{1}{2}[|e\rangle|C_o\rangle + |g\rangle|C_e\rangle] \quad (5)$$

$$|\psi_-\rangle = \frac{1}{2}[|e\rangle|C_e\rangle + |g\rangle|C_o\rangle] \quad (6)$$

The states $|\psi_+\rangle$ and $|\psi_-\rangle$ given by the equations (5) and (6) are the eigenstates of the operator (1) and any given state in a subspace can be expanded as the linear combination

$$|\psi(0)\rangle = c_+|\psi_+\rangle + c_-|\psi_-\rangle \quad (7)$$

in which c_+ and c_- are complex numbers. Operating with the unitary operator, $\hat{U} = e^{i\hat{H}t}$, one can calculate the time evolution of the given state $|\psi(0)\rangle$ as

$$|\psi(t)\rangle = e^{i\hat{H}t}[c_+|\psi_+\rangle + c_-|\psi_-\rangle] \quad (8)$$

From $\hat{H}|\psi_+\rangle = E_+|\psi_+\rangle$ and $\hat{H}|\psi_-\rangle = E_-|\psi_-\rangle$, one obtaines

$$|\psi(t)\rangle = c_+e^{iE_+t}|\psi_+\rangle + c_-e^{iE_-t}|\psi_-\rangle \quad (9)$$

that is a general form of Schrodinger's cat state as a result of the atom-photon interaction in the presence of virtual photons.

3 Examples

Here we give some examples to simply convert the equation (11) to some known cat states.

As the first example, set $c_+ = c_- = \frac{1}{\sqrt{2}}$ in the equation (9) to obtain the initial state

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle + |\psi_-\rangle) \quad (10)$$

Replacing $|\psi_+\rangle$ and $|\psi_-\rangle$ from the equations (4) yields

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|e\rangle + |g\rangle]|\alpha\rangle \quad (11)$$

that is a normalized initial state in which the atom is initially in the mixed state and the field in the coherent state. It is straightforward to obtain the time evolution of this initial state by using the equation (9) as

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{iE_+t}|\psi_+\rangle + e^{iE_-t}|\psi_-\rangle) \quad (12)$$

Some simple calculations after replacing $|\psi_+\rangle$ and $|\psi_-\rangle$ from the equations (4), tends to

$$|\psi(t)\rangle = \frac{2A(t)}{\sqrt{2}}(cos\theta(t)[|g\rangle + |e\rangle]|\alpha\rangle - isin\theta(t)[|g\rangle - |e\rangle]|-\alpha\rangle) \quad (13)$$

in which $A(t) = e^{i(\omega_F|\alpha|^2+\lambda(\alpha+\alpha^*))t}$ and $\theta(t) = \frac{\omega_A t}{2}e^{-2\alpha^2}$. An atomic state measurement in the equation (13) projects the field state into

$$|\psi(t)\rangle = \frac{2A(t)}{\sqrt{2}}(cos\theta(t)|\alpha\rangle \pm isin\theta(t)|-\alpha\rangle) \quad (14)$$

The plus and minus signs appear when the atomic states detected to be in the excited state and the ground state, respectively. The field states $|\alpha\rangle$ and $|-\alpha\rangle$ may be detected with the probabilities $p_+ = cos^2(\frac{\omega_A t}{2}e^{-2\alpha^2})$ and $p_- = sin^2(\frac{\omega_A t}{2}e^{-2\alpha^2})$ respectively. At $t = 0$, one obtains $p_+ = 1$ and $p_- = 0$ that means the initial state is $|\alpha\rangle$. The probabilities p_+ and p_- as functions of α are plotted in the figure 1.

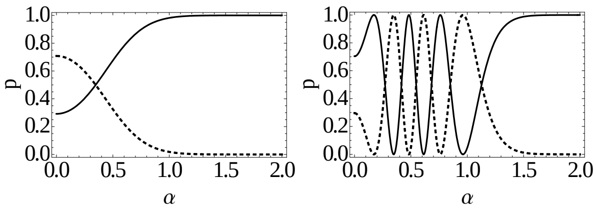

Figure 1: The probabilities of detecting $|\alpha\rangle$ (solid lines) and $|-\alpha\rangle$ (dashed lines) for $\frac{\omega_A t}{2} = 1$, left, and $\frac{\omega_A t}{2} = 10$, right, based on the equation (14).

Based on the figure 1, in small times, the probabilities are aperiodic, while for large times, the probabilities are periodic in some intervals of α. For sufficiently large times, the initial probability values will be restored. As shown in the figure 2, increasing α decreases the frequency of the probabilities oscillations in time.

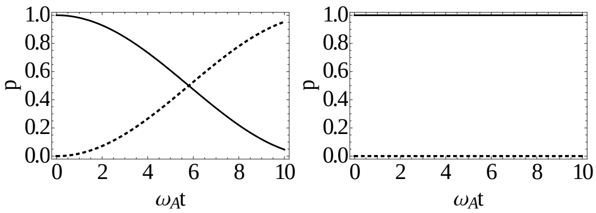

Figure 2: The probabilities of detecting $|\alpha\rangle$ (solid lines) and $|-\alpha\rangle$ (dashed lines) for $\alpha = 1$, left and $\alpha = 2$, right, based on the equation (14).

The light intensity is a parameter by which the frequencies of the probabilities can be controlled to oscillate in time. For $\theta = \pi/2$ the state given by the equation (16) converts to the Yurke-Stoller state

$$|\psi(t)\rangle = A(t)(|\alpha\rangle \pm \mathrm{i}|-\alpha\rangle) \quad (15)$$

as given in [3].

As another example, let $c_+ = \frac{1}{\sqrt{2}}$ and $c_- = -\frac{1}{\sqrt{2}}$ to obtain

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\psi_+\rangle - |\psi_-\rangle) \quad (16)$$

That is equivalent to the atom initially in the mixed state and the field in the coherent state, as

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|g\rangle + |e\rangle]|-\alpha\rangle \quad (17)$$

For witch the time evolution is

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{i\mathrm{E}_+t}|\psi_{1+}\rangle - e^{i\mathrm{E}_-t}|\psi_{1-}\rangle) \quad (18)$$

based on the equation (9). Some simple calculations after replacing $|\psi_+\rangle$ and $|\psi_-\rangle$ from the equations (4), tends to

$$|\psi(t)\rangle = \frac{2A(t)}{\sqrt{2}}(-isin\theta(t)[|g\rangle + |e\rangle]|\alpha\rangle + cos\theta(t)[|g\rangle - |e\rangle]|-\alpha\rangle) \quad (19)$$

An atomic state measurement in the equation (19) projects the field state into

$$|\psi(t)\rangle = \frac{2A(t)}{\sqrt{2}}(-isin\theta(t)|\alpha\rangle \mp cos\theta(t)|-\alpha\rangle) \quad (20)$$

The minus sign appears when excited atom detects and the plus sign appears when detected atom is measured in ground state. The field states $|\alpha\rangle$ and $|-\alpha\rangle$ may be detected with the probabilities $p_+ = sin^2(\frac{\omega_A t}{2}e^{-2\alpha^2})$ and $p_- = cos^2(\frac{\omega_A t}{2}e^{-2\alpha^2})$ respectively. For $\frac{\omega_A t}{2}=0$ the probabilities are $p_+ = 0$ and $p_- = 1$. The results are similar to those of the equation (17) but the probabilities p_+ and p_- are interchanged. For $\theta = \pi/2$ the state given by the equation (20) converts to

$$|\psi(t)\rangle = A(t)(-i|\alpha\rangle \mp |-\alpha\rangle) \quad (21)$$

That is perpendicular to the Yurke-Stoler state introduced in the equation (15).

Another interesting example can be demonstrated by $c_+ = 1$ and $c_- = 0$ to obtain the initial state

$$|\psi(0)\rangle = |\psi_+\rangle \quad (22)$$

That is equivalent to $\frac{1}{\sqrt{2}}[|e\rangle|C_o\rangle + |g\rangle|C_e\rangle]$ that evolves by time according to

$$|\psi(t)\rangle = e^{i\mathrm{E}_+t}|\psi_+\rangle \quad (23)$$

Replacing $|\psi_+\rangle$ from the equations (4) gives

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}e^{i\mathrm{E}_+t}[|e\rangle|C_o\rangle + |g\rangle|C_e\rangle] \quad (24)$$

for which, the atomic state measurement switches the light state to the even or odd cat states with equal probabilities if the measured atomic state appears to be in ground or excited states, respectively.

4 Discussion

Our approach tends to a general superposition of coherent states by taking into account the role of virtual photons in the JCM. The even and odd cat states can be obtained by special selection of the initial states while the previous method needs a dispersive atom-photon interaction before entering the second Ramsey zone. Another striking point of this work is that it generates the well-known Yurke-Stoler states by suitable selection of initial state in the same procedure while it is reported based on the time evolution of the coherent state under a nonlinear Hamiltonian governing an anharmonic oscillator [3]. At last we note that the virtual photons play the role of nonlinear effects in the Yurke-Stoler cat states and the role of the dispersive interaction in the even and odd cat states and unifies the famous cat states in a simple interaction. It is another valuable point that these states are simple examples of the calculated general cat state and it is possible to extract some unknown cat states.

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