**Effect of Positive and Negative Dust Grains on the Modulational Instability of Laser Propagating through Hot Magnetized Plasma**

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Abstract- The present study is aimed to investigate the problem of modulation instability of an intense laser beam in the hot magnetized dusty plasma. The nonlinear propagation of intense circularly-polarized laser beam along the external magnetic field is considered using a relativistic fluid model in the quasi-neutral approximation, which is valid for hot plasma. Nonlinear dispersion equation is obtained. For left- and right-hand polarizations, the growth rate of modulation instability is achieved and the effect of external magnetic field, negative and positive dust grains and kind of polarization on the growth rate is considered.

Keywords: Dusty plasma, Laser interaction, Modulational instability, Nonlinear Schrodinger equation, quasi-neutral
1 Introduction

Modulational instability (MI) is one of the fundamental phenomena in the nonlinear waves theory; the phenomenon that plays major role in different kinds of the nonlinear processes such as envelope solitons, envelope shocks, freak waves, etc. Ponderomotive force originated from the electromagnetic (EM) wave stimulates low frequency perturbations of the electron density; then, they interact with the primary high frequency EM wave in which the amplitude of the pump wave becomes modulated, and the MI of the EM wave occurs. This phenomenon was predicted by Benjamin and Feir [1] for hydrodynamical waves and by Bespalov and Talanov [2] for EM waves in the nonlinear media with a cubic nonlinearity. The examples of MI from water wave hydrodynamics, electrodynamics, nonlinear optics, and convection theory can be found in Ref. 3. The MI of a laser pulse in the cold plasma has been studied in several works [4]. Already, effect of temperature on the MI in quasi-neutral plasma has been investigated by Sepehri Javan [5]. In this work, we have studied the effect of positive and negative dust grains on the MI.

2 Deriving Nonlinear Wave Equation

We consider the propagation of circularly polarized EM wave along the external magnetic field \( B_0 = B_0 \hat{e}_z \) in the hot dusty plasma with . From Maxwells equation, we can write wave equation as

\[
\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \frac{4\pi}{c} \mathbf{J}, \tag{1}
\]

where \( A \) is the vector potential, \( c \) is the speed of light, \( \mathbf{J} = -n_e e \mathbf{v}_e \) is the current density of electrons of plasma, \( n_e, e, \) and \( \mathbf{v}_e \) are the density, charge and velocity of electron, respectively. Now, we write the relativistic fluid momentum equation for electrons

\[
\frac{\partial \mathbf{v}_e}{\partial t} + \frac{\mathbf{v}_e \cdot \nabla \mathbf{v}_e}{\gamma m_0 c} = \frac{e}{c} \mathbf{A} + e \nabla \varphi - \frac{e}{m_0 c} \mathbf{p} \times \mathbf{A} - k_B T_e \nabla \ln n_e \mathbf{v}_e, \tag{2}
\]

where \( \mathbf{p}, \gamma = (1 + p^2 / m_0^2 c^2)^{1/2}, m_0, \varphi, \omega_e = eB_0 / m_0 c, k_B \) and \( T_e \) are momentum, relativistic Lorentz factor, ambipolar scalar potentials, cyclotron frequency, rest mass of electron, the Boltzmann constant and temperature of electron, respectively. We consider the vector potential of laser wave as following

\[
A = \frac{1}{2} \tilde{A} \left( \hat{e}_x + i \sigma \hat{e}_y \right) \exp(-i\omega_0 t + ik_0 z) + c.c. \tag{3}
\]

where \( \omega_0, k_0 \) are the frequency and wave number, \( \sigma = +1,-1 \) denotes the right- and left-hand circularly polarized wave, and \( \tilde{A}(z,t) \) is the slowly varying amplitude. Inserting Eq. (3) into Eq. (2), we can find that [5]

\[
\mathbf{p} = (1 - \sigma\alpha / \gamma)^{-1} \tilde{A}, \tag{4}
\]

\[
\{ \nabla [\Phi - \beta_e (\gamma + \frac{\sigma \alpha}{2\gamma^2}) - \ln(n_{oe}^{-1})] \} \cdot \mathbf{v}_e = 0, \tag{5}
\]

where \( \mathbf{p} = p / m_0 c, \tilde{A} = e\mathbf{A} / m_0 c^2, \alpha = \omega_e / \omega_0, \Phi = e\varphi / k_B T_e, \beta_e = c^2 / V_e^2, V_e^2 = k_B T_e / m_0 \) and \( n_{oe} \) is unperturbed electron density. Integrating Eq. (5), we write

\[
n_e = n_{oe} \exp \{ -\beta_e (\gamma + 1 - \sigma\alpha) |\mathbf{p}|^2 / 2\gamma^2 \}. \tag{6}
\]

Assuming isothermal equation of state for positive and negative dust grains and ions we obtain [5]

\[
n_i = n_{oi} \exp(-e\varphi / k_B T_i), \tag{7}
\]

\[
n_{d-} = n_{d0-} \exp(z_e e\varphi / k_B T_0), \tag{8}
\]

\[
n_{d+} = n_{d0+} \exp(-z_e e\varphi / k_B T_0), \tag{9}
\]

where \( z_e, z_\alpha \) are ionization degree of positive and negative dust grains. In the quasi-neutral regime \( n_s n_{d+} + n_i - n_e - z_{d-} = 0 \) and weakly relativistic laser \( \gamma \approx 1 + |\mathbf{p}|^2 / 2 \), we achieve

\[
n_e = n_{e0} \exp\{ -\mu (\gamma - 1 - \sigma\alpha) |\mathbf{p}|^2 / 2\gamma^2 \}. \tag{10}
\]

\[
\mu = \beta_e (1 + \frac{1}{\delta_i / S_i + \delta_0 / \delta_+ / S_0} + \delta_+ / S_+)^{-1}, \tag{11}
\]

\[
\delta_i = T/\gamma T^{-1} - S_i = n n_{oe}^{-1}, \delta_+ = T/\gamma T^{-1} - S_+, \delta_0 = T/\gamma T^{-1} - S_0 \tag{12}
\]

In physical units, from Eq. (4) for the velocity of electrons we can obtain

\[
\mathbf{v}_e = (\gamma - \sigma\alpha)^{-1} c \tilde{A}. \tag{12}
\]

Also, for Lorentz factor we can approximately write \( \gamma \approx 1 + (1 - \sigma\alpha)^2 |\mathbf{A}|^2 / 2 \). Taking into account Eqs. (10) and (12) we have

\[
\mathbf{J} \approx -\frac{4\pi e^2}{c} A P \exp(-\mu(1 - \sigma\alpha)^2 |\mathbf{A}|^2 / 2), \tag{13}
\]
where \( \omega_p^2 = 4\pi n_0 e^2 / m_0 \) and 
\[ P = (1 - \alpha \alpha)^{-1} \left[ 1 - (1 - \alpha \alpha)^{-3} | \mathbf{A} |^2 / 2 \right]. \]
By introducing the following dimensionless variables 
\[ \tau = \frac{\omega_p^2 t}{\omega_0 c}, \quad U_g = \frac{\omega_0 v_g}{c}, \quad \xi = \frac{\omega_p z}{c} + U_g \tau, \]
and replacing Eqs. (3) and (13) in the Eq. (1) we find
\[ \frac{\partial \alpha}{\partial \tau} + \frac{1}{2} \frac{\partial^2 \alpha}{\partial \xi^2} + Da = 0, \tag{14} \]
where \( v_g = k_0 c^2 / \omega_0 \) is the group velocity and 
\[ D = \frac{1 - \exp\left(\frac{\mu_0^2 A^2}{\alpha \alpha} - 1\right)(1 - (1 - \alpha \alpha)^{-3}|A|^2 / 2)}{2(1 - \alpha \alpha)}. \]

3 Modulational Instability

To derive the dispersion relation for MI, we use the well-known method in which we suppose 
\[ a = (a_0 + a_1) \exp(i\Lambda \tau), \tag{15} \]
where \( a_0 \) is a real parameter, \( a_0 > |a_1| \), and 
\[ \Lambda = \frac{1}{2 - 1 - \alpha \alpha} \left( 1 - \exp\left(\frac{\mu_0^2 A^2}{\alpha \alpha} - 1\right)(1 - \frac{A_0^2}{1 - \alpha \alpha}) \right). \tag{16} \]
By assuming \( a_1 = X + iY \), substituting Eq. (15) in Eq. (14) and linearizing it with respect to we find
\[ 2 \frac{\partial^2 Y}{\partial \tau^2} - \frac{\partial^2 X}{\partial \xi^2} - \frac{\mu_0^2 A^2}{1 - \alpha \alpha} \exp\left(-\frac{\mu_0^2 A^2}{1 - \alpha \alpha}\right) \] 
\[ \times \left( 1 - \mu_0^2 A^2 / 2(1 - \alpha \alpha) \right) X = 0, \tag{17} \]
\[ \frac{\partial Y}{\partial \tau} + 2 \frac{\partial^2 Y}{2 \partial \xi^2} = 0. \tag{18} \]
Supposing \( X = \tilde{X} e^{-i\alpha_1 \tau + iK_1 \xi}, Y = \tilde{Y} e^{-i\alpha_1 \tau + iK_1 \xi} \), Eqs. (17) and (18) lead to following dispersion relation
\[ \Omega^2 = -\frac{\mathcal{K}^2}{2} \left[ \frac{\mu_0^2 A^2}{(1 - \alpha \alpha)^4} \right] \left( 1 + \mu_0(1 - \alpha \alpha) - \frac{\mu_0^2 A^2}{2(1 - \alpha \alpha)} \right) \]
\[ \times \exp\left( -\frac{\mu_0^2 A^2}{2(1 - \alpha \alpha)} - \frac{1}{2} \mathcal{K}^2 \right), \tag{19} \]
by which we can extract the MI growth rate 
\[ \Gamma = -i\Omega \] as below

\[ \Gamma = \frac{K}{\sqrt{2}} \left[ \frac{a_0^2 / 2}{(1 - \alpha \alpha) ^2} \left( \frac{1}{(1 - \alpha \alpha)^2} + \mu_0 \frac{a_0^2}{2(1 - \alpha \alpha)} - \frac{\mu_0^2 A^2}{2(1 - \alpha \alpha)} \right) \right] \]
\[ \times \exp\left( -\frac{\mu_0^2 A^2}{2(1 - \alpha \alpha)} - \frac{1}{2} \mathcal{K}^2 \right). \tag{20} \]
We can see that the growth rate has a maximum
\[ \Gamma_{\max} = \frac{a_0^2 / 4}{(1 - \alpha \alpha)^2} \left( \frac{1}{(1 - \alpha \alpha)^2} + \mu_0 \frac{a_0^2}{2(1 - \alpha \alpha)} \right) \]
\[ \times \exp\left( -\frac{\mu_0^2 A^2}{2(1 - \alpha \alpha)} - \frac{1}{2} \mathcal{K}^2 \right). \tag{21} \]

4 Numerical Discussions

We have supposed Nd:YAG laser with frequency \( \omega_0 = 1.88 \times 10^{15} \text{s}^{-1}, \quad a_0 = 0.1 \). For plasma species we consider identical temperature \( T = 5 \text{keV} \) also for ionization degree \( z_+ = z_- = 100 \). For numerical purposes we introduce new parameters \( n_{0d+} + n_{0i} = n_{0d-} + n_{0e} = n_0, \quad n_0 = \beta n_0, \tag{22} \)
\( n_{0d-} = (1 - \eta) n_0, \quad n_{0d+} = (1 - \beta) n_0. \tag{23} \)
Figure (1) shows variation of growth rate with respect to \( K \) for an electron gas in the absence of dust grains. We can see that magnetization of plasma leads to the increase in the growth rate for

![Figure 1](image-url)
Conclusions

We have investigated the MI of short laser pulse in hot magnetized dusty plasma. Effect of external magnetic field, state of polarization and dust grains on the instability have been studied. It is found that using negative dust grains besides electrons enhances the MI growth rate.

References