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تأثیر دانه‌های مثبت و منفی غبار بر ناپایداری مدوله ای لیزر منتشره در پلاسمای داغ مغناطیده

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چکیده – هدف از این مطالعه، تحقیق مسئله ناپایداری مدولاسیون پالس لیزر در پلاسمای داغ مغناطیده غباری می‌باشد. انتشار غیر خطی باریکه لیزر شدید قطبیده دایروی در امتداد میدان مغناطیسی خارجی با استفاده از مدل سیالی نسبیتی در تقریب شبه-خنثی که برای پلاسمای داغ معتبر است، در نظر گرفته شده است. معادله پاشندگی غیرخطی بدست آمده است. برای قطبشهای راستگرد و چپگرد نرخ رشد ناپایداری مدوله‌ای حاصل شده و تاثیر میدان مغناطیسی خارجی، دانه‌های غبار منفی و مثبت و نیز نوع قطبش بر نرخ رشد بررسی شده است.

کلید واژه-پلاسمای غباری، اندرکنش لیزر، معادله ی غیر خطی شرودینگر، ناپایداری مدولاسیون، شبه خنثی

Effect of Positive and Negative Dust Grains on the Modulational Instability of Laser Propagating through Hot Magnetized Plasma

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Abstract- The present study is aimed to investigate the problem of modulation instability of an intense laser beam in the hot magnetized dusty plasma. The nonlinear propagation of intense circularly-polarized laser beam along the external magnetic field is considered using a relativistic fluid model in the quasi-neutral approximation, which is valid for hot plasma. Nonlinear dispersion equation is obtained. For left- and right-hand polarizations, the growth rate of modulation instability is achieved and the effect of external magnetic field, negative and positive dust grains and kind of polarization on the growth rate is considered.

Keywords: Dusty plasma, Laser interaction, Modulational instability, Nonlinear Schrodinger equation, quasi-neutral

1 Introduction

Modulational instability (MI) is one of the fundamental phenomena in the nonlinear waves theory; the phenomenon that plays major role in different kinds of the nonlinear processes such as envelope solitons, envelope shocks, freak waves, etc. Pondermotive force originated from the electromagnetic (EM) wave stimulates low frequency perturbations of the electron density; then, they interact with the primary high frequency EM wave in which the amplitude of the pump wave becomes modulated, and the MI of the EM wave occurs. This phenomenon was predicted by Benjamin and Feir [1] for hydrodynamical waves and by Bespalov and Talanov [2] for EM waves in the nonlinear media with a cubic nonlinearity. The examples of MI from water wave hydrodynamics, electrodynamics, nonlinear optics, and convection theory can be found in Ref. 3. The MI of a laser pulse in the cold plasma has been studied in several works [4]. Already, effect of temperature on the MI in quasi-neutral plasma has been investigated by Sepehri Javan [5]. In this work, we have studied the effect of positive and negative dust grains on the MI.

2 Deriving Nonlinear Wave Equation

We consider the propagation of circularly polarized EM wave along the external magnetic field $B_0 = B_0\hat{e}_z$ in the hot dusty plasma with . From Maxwells equation, we can write wave equation as

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2 \mathbf{A} = \frac{4\pi}{c}\mathbf{J}, \tag{1}$$

where $\mathbf{A}$ is the vector potential, c is the speed of light, $\mathbf{J} = -n_e e \mathbf{v_e}$ is the current density of electrons of plasma, n_e, e, and $\mathbf{v}_e$ are the density, charge and velocity of electron, respectively. Now, we write the relativistic fluid momentum equation for electrons

$$\begin{aligned}\frac{\partial \mathbf{p}}{\partial t} + \frac{(\mathbf{p}.\nabla)\mathbf{p}}{\gamma m_0} &= \frac{e}{c}\frac{\partial \mathbf{A}}{\partial t} + e\nabla\varphi \\ &- \frac{e\mathbf{p}\times\nabla\times\mathbf{A}}{\gamma m_0 c} - \frac{\omega_c}{\gamma}\mathbf{p}\times\hat{\mathbf{e}}_\mathbf{z} - k_B T_e \nabla \ln n_e\end{aligned}, \tag{2}$$

where $\mathbf{p}$, $\gamma = (1 + p^2/m_0^2c^2)^{1/2}$, m_0, φ, $\omega_c = eB_0/m_0c$, k_B and T_e are momentum, relativistic Lorentz factor, ambipolar scalar potentials, cyclotron frequency, rest mass of electron, the Boltzmann constant and temperature of electron, respectively. We consider the vector potential of laser wave as following

$$\mathbf{A} = \frac{1}{2}\tilde{A}(\hat{\mathbf{e}}_\mathbf{x} + i\sigma\hat{\mathbf{e}}_\mathbf{y})\exp(-i\omega_0 t + ik_0 z) + c.c.\ , \tag{3}$$

where ω_0, k_0 are the frequency and wave number, $\sigma = +1,-1$ denotes the right- and left-hand circularly polarized wave, and $\tilde{A}(z,t)$ is the slowly varying amplitude. Inserting Eq. (3) into Eq. (2), we can find that [5]

$$\overline{\mathbf{p}} = (1 - \sigma\alpha/\gamma)^{-1}\overline{\mathbf{A}}, \tag{4}$$

$$\{\vec{\nabla}[\Phi - \beta_e(\gamma + \frac{\sigma\alpha}{2\gamma^2}) - \ln(\frac{n_e}{n_{0e}})]\}.\hat{e}_z = 0, \tag{5}$$

where $\overline{p} = p/m_0c$, $\overline{A} = eA/m_0c^2$, $\alpha = \omega_c/\omega_0$, $\Phi = e\varphi/k_BT_e$, $\beta_e = c^2/V_{Te}^2$, $V_{Te}^2 = k_\mathrm{B}T_e/m_0$ and n_{0e} is unperturbed electron density. Integrating Eq. (5), we can write

$$n_e = n_{0e}\exp\{\Phi - \beta_e[\gamma - 1 - \sigma\alpha|\overline{\mathbf{p}}|^2/2\gamma^2]\}. \tag{6}$$

Assuming isothermal equation of state for positive and negative dust grains and ions we obtain [5]

$$n_i = n_{0i}\exp(-e\varphi/k_BT_i), \tag{7}$$

$$n_{d-} = n_{0d-}\exp(z_- e\varphi/k_BT_-), \tag{8}$$

$$n_{d+} = n_{0d+}\exp(-z_+ e\varphi/k_BT_+), \tag{9}$$

where z_+, z_- are ionization degree of positive and negative dust grains. In the quasi-neutral regime $z_+n_{d+} + n_i - n_e - z_-n_{d-} = 0$ and weakly relativistic laser $\gamma \approx 1 + |\overline{\mathbf{P}}|^2/2$, we achieve

$$n_e = n_0\exp[-\mu(\gamma - 1 - \sigma\alpha|\overline{\mathbf{p}}|^2/2\gamma^2)], \tag{10}$$

$$\mu = \beta_e(1 + \frac{1}{\delta_i/S_i + \delta_-/S_- + \delta_+/S_+})^{-1}, \tag{11}$$

$$\delta_i = T_iT_e^{-1}, S_i = n_in_{0e}^{-1}, \delta_+ = T_{d+}T_e^{-1},$$

$$\delta_- = T_{d-}T_e^{-1}, S_+ = n_{d+}z_+^2n_{0e}^{-1}, S_- = n_{d-}z_-^2n_{0e}^{-1}.$$

In physical units, from Eq. (4) for the velocity of electrons we can obtain

$$\mathbf{v}_e = (\gamma - \sigma\alpha)^{-1}c\overline{\mathbf{A}}. \tag{12}$$

Also, for Lorentz factor we can approximately write $\gamma \approx 1 + (1-\sigma\alpha)^{-2}|\overline{\mathbf{A}}|^2/2$. Taking into account Eqs. (10) and (12) we have

$$\mathbf{J} \approx \frac{-4\pi\omega_p^2}{c}\mathbf{A}P\exp(-\mu(1-\sigma\alpha)^{-1}\left|\overline{\mathbf{A}}\right|^2/2), \tag{13}$$

where $\omega_p^2 = 4\pi n_{0e} e^2 / m_0$ and $P = (1-\sigma\alpha)^{-1}\left(1-(1-\sigma\alpha)^{-3}\,|\,\overline{\mathbf{A}}\,|^2\,/2\right)$. By introducing the following dimensionless variables

$\tau = \dfrac{\omega_p^2}{\omega_0} t,\ \ U_g = \dfrac{\omega_0}{\omega_p}\dfrac{v_g}{c},\ \ \xi = \dfrac{\omega_p}{c} z + U_g \tau$, and replacing Eqs. (3) and (13) in the Eq. (1) we find

$$i\frac{\partial a}{\partial \tau} + \frac{1}{2}\frac{\partial^2 a}{\partial \xi^2} + Da = 0., \tag{14}$$

where $v_g = k_0 c^2 / \omega_0$ is the group velocity and

$$D = \frac{1 - \exp\left(\dfrac{\mu |A|^2 / 2}{\sigma\alpha - 1}\right)(1 - (1-\sigma\alpha)^{-3} |A|^2 / 2)}{2(1-\sigma\alpha)}.$$

3 Modulational Instability

To derive the dispersion relation for MI, we use the well-known method in which we suppose

$$a = (a_0 + a_1)\exp(i\Lambda\tau), \tag{15}$$

where a_0 is a real parameter, $a_0 >> | a_1 |$, and

$$\Lambda = \frac{1/2}{1-\sigma\alpha}\left(1 - \exp\left(\frac{a_0{}^2 \mu / 2}{\sigma\alpha - 1}\right)\left(1 - \frac{a_0{}^2/2}{(1-\sigma\alpha)^3}\right)\right). \tag{16}$$

By assuming $a_1 = X + iY$, substituting Eq. (15) in Eq. (14) and linearizing it with respect to we find

$$2\frac{\partial Y}{\partial \tau} - \frac{\partial^2 X}{\partial \xi^2} - \frac{a_0^2}{(1-\sigma\alpha)^2}\exp\left(-\frac{\mu a_0^2/2}{(1-\sigma\alpha)}\right) \times \left(\frac{1}{(1-\sigma\alpha)^2} + \mu\left(1 - \frac{a_0{}^2}{2(1-\sigma\alpha)^3}\right)\right) X = 0, \tag{17}$$

$$\frac{\partial X}{\partial \tau} + \frac{1}{2}\frac{\partial^2 Y}{\partial \xi^2} = 0\ . \tag{18}$$

Supposing $X = \tilde{X} e^{-i\Omega\tau + iK\zeta},\ Y = \tilde{Y} e^{-i\Omega\tau + iK\zeta}$, Eqs. (17) and (18) lead to following dispersion relation

$$\Omega^2 = -\frac{K^2}{2}\left\{\frac{a_0^2/2}{(1-\sigma\alpha)^4}\left(1 + \mu(1-\sigma\alpha) - \frac{\mu a_0{}^2/2}{1-\sigma\alpha}\right) \times \exp\left(-\frac{\mu}{2}\frac{a_0^2}{(1-\sigma\alpha)}\right) - \frac{1}{2}K^2\right\}, \tag{19}$$

by which we can extract the MI growth rate $\Gamma = -i\Omega$ as below

$$\Gamma = \frac{K}{\sqrt{2}}\left[\frac{a_0^2/2}{(1-\sigma\alpha)^2}\left(\frac{1}{(1-\sigma\alpha)^2} + \mu\left(1 - \frac{a_0{}^2/2}{(1-\sigma\alpha)^3}\right)\right) \times \exp\left(\frac{\mu a_0^2/2}{\sigma\alpha - 1}\right) - \frac{1}{2}K^2\right]^{\frac{1}{2}}. \tag{20}$$

We can see that the growth rate has a maximum

$$\Gamma_{\max} = \frac{a_0^2/4}{(1-\sigma\alpha)^2}\left[\frac{1}{(1-\sigma\alpha)^2} + \mu\left(1 - \frac{a_0{}^2/2}{(1-\sigma\alpha)^3}\right)\right] \times \exp\left(\frac{\mu a_0^2/2}{\sigma\alpha - 1}\right). \tag{21}$$

4 Numerical Discussions

We have supposed Nd:YAG laser with frequency $\omega_0 = 1.88 \times 10^{15}\,\mathrm{s}^{-1}$, $a_0 = 0.1$. For plasma species we consider identical temperature $T = 5keV$ also for ionization degree $z_+ = z_- = 100$. For numerical purposes we introduce new parameters

$$n_{0d+} + n_{0i} = n_{0d-} + n_{0e} = n_0,\ \ n_{0i} = \beta n_0, \tag{22}$$

$$n_{0d-} = (1-\eta) n_0,\ \ n_{0d+} = (1-\beta) n_0. \tag{23}$$

Figure (1) shows variation of growth rate with respect to K for an electron gas in the absence of dust grains. We can see that magnetization of plasma leads to the increase in the growth rate for

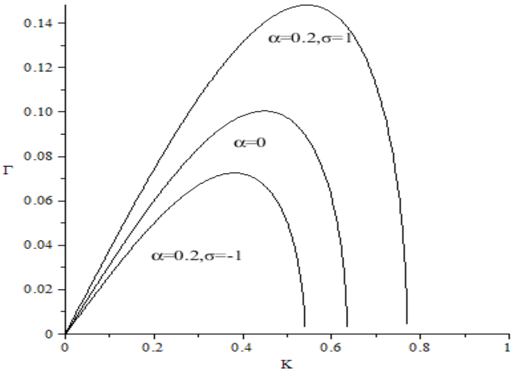

Figure 1: Γ vs. K, $\beta = \eta = 1$ (electron gas plasma), $n_{0e} = n_{0i} = 10^{12}\,cm^{-3}$.

right-hand polarization and decrease in it for left-hand one. In figure (2) we set parameters so that in the same total density $n_0 = 10^{12}\,cm^{-3}$ there are only electron and positive dust grains. We can see that the existence of heavy and multiple-ionized positive dust grains instead of single ionized ions, causes increase in the growth rate.

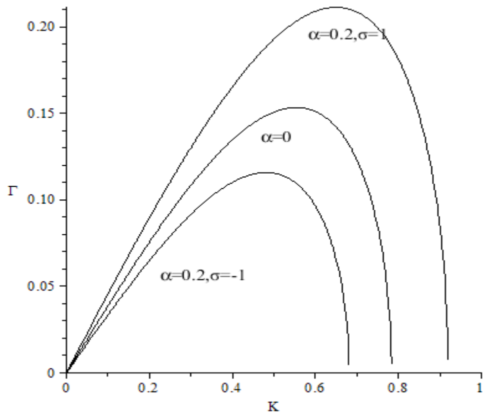

Figure 2: Γ vs. K, $\beta = 1, \eta = 0$ (electron-positive dust), $n_{0e} = n_0 = 10^{12} cm^{-3}$.

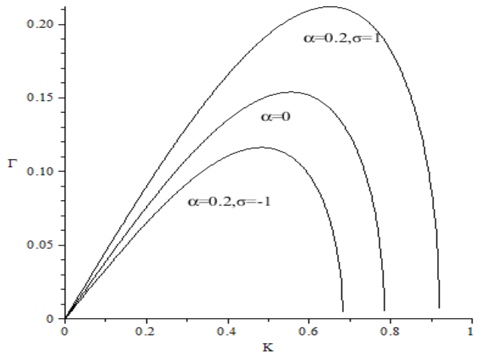

Figure 3: Γ vs. K, $\beta = 0.25, \eta = 0.75$ (electron-positive and negative dust), $n_{0e} = 10^9 cm^{-3}$.

In figure (3) there is electron-positive and negative dust grains plasma, however the density of negative dust magnificently is more than positive one. We can find out that the admixture of negative grains does not sufficiently effect on the maximum of normalized growth rate in comparison with previous case. In figure (3) we have plotted the maximum growth rate as a function of α for the case corresponding to the previous case of four-component dusty plasma. As we see, increase in the magnetic field causes considerable increase of growth rate in right-hand mode and its decrease in the left-hand one.

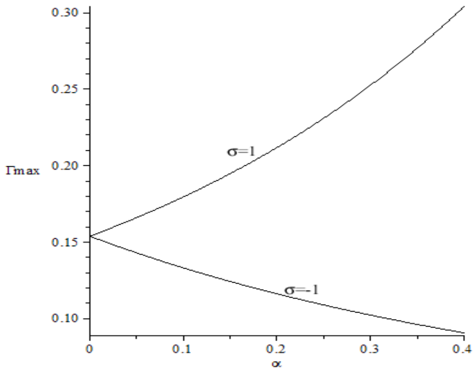

Figure 3: $\Gamma_{\max}$ vs. α, $\beta = 0.25, \eta = 0.75$ (electron-positive and negative dust), $n_{0e} = 10^9 cm^{-3}$.

5 Conclusions

We have investigated the MI of short laser pulse in hot magnetized dusty plasma. Effect of external magnetic field, state of polarization and dust grains on the instability have been studied. It is found that using negative dust grains besides electrons enhances the MI growth rate.

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