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مدهای غیرخطی امواج الکترومغناطیسی منتشر شونده در آرایه‌ی تناوبی و مغناطیده از نانوذرات

ناصر سپهری جوان و حدیث هاشمی

دانشگاه محقق اردبیلی، دانشکده علوم پایه، گروه فیزیک

چکیده – انتشار غیرخطی یک موج الکترومغناطیسی خودهدایتگر پرشدت در یک آرایه تناوبی از نانوذرات مطالعه شده است. با استفاده از روش اختلال، معادله غیرخطی توصیف کننده‌ی اندرکنش نانوذرات با لیزر در رژیم نسبیتی ضعیف بدست آمده است. ویژه مدهای عرضی، رابطه‌ی پاشندگی غیرخطی و سرعت گروه حاصل شده اند. تاثیر دامنه‌ی لیزر، اندازه و فاصله‌ی نانوذرات، شدت میدان مغناطیسی خارجی و قطبش لیزر روی پاشندگی غیرخطی و نمایه‌ی دامنه بررسی شده است.

کلید واژه- آرایه‌ی تناوبی مغناطیده، اندرکنش با لیزر، پاشندگی غیرخطی، مدهای عرضی، نانوذرات.

Nonlinear Modes of Electromagnetic Waves Propagating Through Magnetized Periodic Nano-Particles Lattice

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Abstract- Self-guided nonlinear propagation of an intense electromagnetic wave in a magnetized periodic array of nanoparticles is studied. Using a perturbative method, nonlinear equation describing laser-nanoparticles interaction in the weakly relativistic regime is derived. Transverse Eigen modes, nonlinear dispersion relation and group velocity are obtained. Effect of the amplitude of laser, separation and size of nanoparticles, the intensity of the external magnetic field and the laser polarization on the nonlinear dispersion and amplitude profile are investigated.

Keywords: Laser interaction, Magnetized periodic array, Nanoparticles, Nonlinear dispersion, Transverse Eigen modes.

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1 Introduction

During interaction of laser beams with nanoparticles array, parametric scattering of laser can produce unstable electromagnetic (EM) radiation with lower frequency by excitation of plasmonic or acoustic collective vibrations via parametric Raman [1] or Brillouin [2] instabilities. Nonlinear interaction of laser with one-dimensional periodically nanostructured metals can lead to the localization of light in a region with characteristic dimensions smaller than the incident wavelength. This phenomenon called nano-focusing. Such a nano-focusing property is observed theoretically and experimentally in the interaction of laser with determinate numbers or chain of nanoparticles [3, 4]. A well-known nonlinear optical phenomenon that appears via the change of the medium refractive index exposed to the intense EM radiation is Self-focusing [5]. There is an effective distance in which focusing balances with diffraction and beam travels without any convergence. This situation called self-trapping or self-guiding regime. In this paper we investigate propagation of an intense laser beam in a magnetized periodic array of nanoparticles in the self-guiding regime.

2 Motion Equations

Consider a nanoparticles lattice with radius r_0, separation d and electronic density n_0. An (EM) wave propagates along an external magnetic field $B_0\hat{\mathbf{e}}_\mathbf{z}$. We consider the EM fields as following

$$\mathbf{E} = \hat{E}[\cos(kz-\omega t)\hat{\mathbf{e}}_\mathbf{x} - \sigma \sin(kz-\omega t)\hat{\mathbf{e}}_y], \quad (1)$$

$$\mathbf{B} = (c/\omega)\mathbf{k}\times\mathbf{E}, \quad (2)$$

where, $\hat{E},\omega,k,c$ are the slowly varying amplitude, frequency, wave number and light speed, respectively. Also $\sigma=+1,-1$ denote right- and left-hand polarizations, respectively. We assume that ions are immobile. To solve the motion equation for electronic cloud we use the perturbative method. For Lorentz factor we have

$$\gamma^{(0)}=1,\ \ \gamma^{(1)}=0,\ \ \gamma^{(2)}=(v^{(1)}/c)^2, \quad (3)$$

where superscript (n) refers to the nth order perturbed parameters. For electronic cloud displacement and velocity we consider $\mathbf{r}=\mathbf{r}^{(1)}+\mathbf{r}^{(2)}+\mathbf{r}^{(3)}$, $\mathbf{v}=\mathbf{v}^{(1)}+\mathbf{v}^{(2)}+\mathbf{v}^{(3)}$. Electric and magnetic fields of the laser beam are of the first order, obviously. Displacement of electronic cloud results a restoration force $(-\omega_p/\sqrt{3})\mathbf{r}$, where $\omega_{pe}^2 = 4\pi e^2 n_0/m_0$, e and m_0 are the electron charge and mass, respectively. The electron cloud motion equation is

$$\frac{d}{dt}(\gamma\mathbf{v}) + \frac{\omega_{pe}^2}{3}\mathbf{r} = \frac{-e}{m_0}\mathbf{E} - \frac{e}{m_0 c}\mathbf{v}\times(\mathbf{B}+\mathbf{B}_0). \quad (4)$$

Solving the first order equation leads to following

$$x^{(1)} = \frac{-e\hat{E}/m_0}{(\omega_{pe}^2/3)-\omega^2+\sigma\omega\omega_c}\cos(kz-\omega t), \quad (5)$$

$$y^{(1)} = \frac{-\sigma e\hat{E}/m_0}{(\omega_{pe}^2/3)-\omega^2+\sigma\omega\omega_c}\sin(kz-\omega t), \quad (6)$$

and $z^{(1)}=0$, where $\omega_c = eB_0/m_0 c$. The second order motion equation can be derived as below

$$\frac{d\mathbf{v}^{(2)}}{dt} + \frac{\omega_{pe}^2\mathbf{r}^{(2)}}{3} = \frac{-e}{m_0 c}[\mathbf{v}^{(1)}\times\mathbf{B} + \mathbf{v}^{(2)}\times\mathbf{B}_0], \quad (7)$$

which leads to $\mathbf{r}^{(2)}=0$. Finally, with the same manner, third order motion equation results

$$x^{(3)} = -\frac{a^3\omega^7 c\cos(kz-\omega t)}{2[(\omega_{pe}^2/3)-\omega^2+\sigma\omega\omega_c]^4}, \quad (8)$$

$$y^{(3)} = -\frac{a^3\omega^7 c\sigma\sin(kz-\omega t)}{2[(\omega_{pe}^2/3)-\omega^2+\sigma\omega\omega_c]^4}, \quad (9)$$

and $z^{(3)}=0$, where $a = e\hat{E}/(m_0\omega c)$.

3 Transverse Amplitude Profile and Nonlinear Dispersion Relation

In order to obtain the nonlinear dispersion relation, we need to find the nonlinear current density $\mathbf{J} = -4\pi e n_0 l \mathbf{v}/3$, where $l = (r_0/d)^3$. Using Maxwell equations results the following wave equation

$$\nabla^2 \mathbf{E} - \nabla(\nabla.\mathbf{E}) = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}. \qquad (10)$$

By constituting current density and cylindrical symmetry of amplitude we obtain

$$\frac{\partial^2 a}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial a}{\partial \bar{r}} = (\bar{k}^2 - \varpi^2)a - \frac{4\pi a l \varpi^2}{1 - 3\varpi^2 + 3\sigma\varpi\varpi_c} \times \{1 + \frac{a^2\varpi^2/2}{[(1/3) - \varpi^2 + \sigma\varpi\varpi_c]^3}\}, \qquad (11)$$

where $k\omega_{pe}/c = \bar{k}$, $\omega/\omega_{pe} = \varpi$, $\omega_c/\omega_{pe} = \varpi_c$ and $r\omega_{pe}/c = \bar{r}$. Integrating the both sides of equation (11) with respect to $\bar{r}$ and using the following boundary conditions

$$\begin{cases} da/d\bar{r} = 0 \ \& \ a = a_0 \ \ at \ \ \bar{r} = 0 \\ da/d\bar{r} = 0 \ \& \ a = 0 \ \ at \ \ \bar{r} \to \infty \end{cases}, \qquad (12)$$

lead to the following nonlinear dispersion

$$\frac{-4\pi l\varpi^2/3}{1/3 - \varpi^2 + \sigma\varpi\varpi_c}\{1 - \frac{a_0^2\varpi^2/2}{(1/3 - \varpi^2 + \sigma\varpi\varpi_c)^3}\} \varpi^2 - \bar{k}^2 + \frac{2}{a_0^2}\int_0^\infty \frac{1}{\bar{r}}(\frac{\partial a}{\partial \bar{r}})^2 d\bar{r} = 0. \qquad (13)$$

Figure (1) shows the nonlinear dispersion curves for different values of l where $a_0 = 0.05$ and $\varpi_c = 0.4$. It can be seen that larger l have the greater cut-off frequency. Moreover there are two possible frequencies for propagation of the EM modes for any values of wave number. The lower (low-frequency) branches asymptotically approach to the $\omega \approx 0.81$ for right-hand and $\omega \approx 0.41$ for left-hand waves. The upper branch waves (high-frequency) asymptotically approach to $\omega = kc$ at large ω and k. Dispersion curves of EM waves with right-hand polarization are placed higher than left-hand ones.

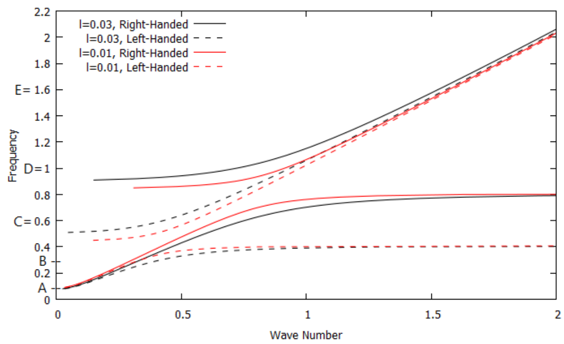

Fig. 1 Dispersion curves, $a_0 = 0.05$ and $\varpi_c = 0.4$.

Figure (2) has been plotted to investigate the laser intensity effect on the dispersion curves. It can be seen that in identical situations, the wave with higher intensity can propagate with lower frequencies. The more noticeable fact is that the lower branch corresponds to the left-hand wave with very small intensity starts from a point near its asymptote. This means that the group velocity of such a curve is near zero from the beginning.

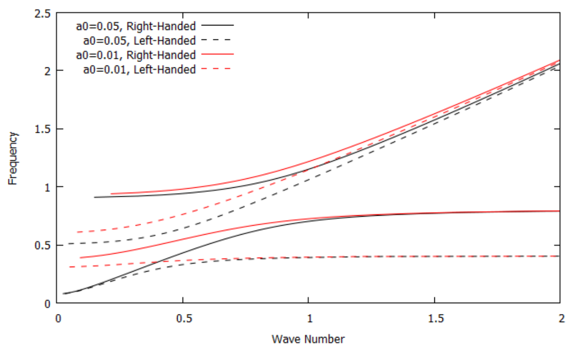

Fig. 2 Dispersion curves, $l = 0.03$ and $\varpi_c = 0.4$.

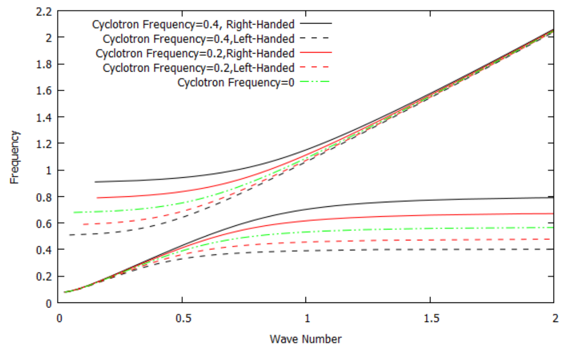

Fig. 3 Dispersion curves, $a_0 = 0.05$ and $l = 0.03$.

In Fig. (3) effect of external magnetic field on dispersion curves has been studied. Increase in the external magnetic field causes the up-shift of frequency for right-hand mode and its down-shift for left-hand one. The group velocity of propagating EM waves can be found with numerical differentiation from the dispersion relation. Figure (4) presents group velocities normalized by the light speed for dispersion curves corresponding to Fig. (1). As it can be seen the lower branches of dispersion curves start from a value far from zero; reach a maximum value and then fall to zero asymptotically. But the upper branches start from low values and grow to light speed. Left-hand polarized waves tend to asymptotes more rapidly than right-hand ones. However, right-hand waves have maximum values higher than left-hand ones. In addition, lower values of l cause the higher group velocities.

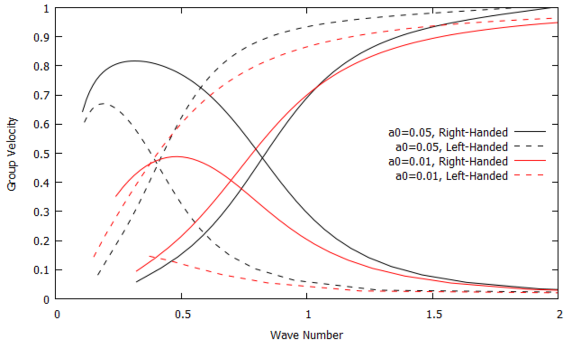

Fig. 4 Group velocity, $a_0 = 0.05$ and $\overline{\omega}_c = 0.4$.

In order to study the effect of the magnetic field on the group velocity, we have plotted Fig. (5). For lower branches, increase in the external magnetic field leads to the increase in the group velocity of the right-hand polarized waves and it inversely acts to left-hand ones. For upper branches, scenario changes and increase in the magnetic field results to the decrease of group velocity for right-hand modes its increase for left-hand ones.

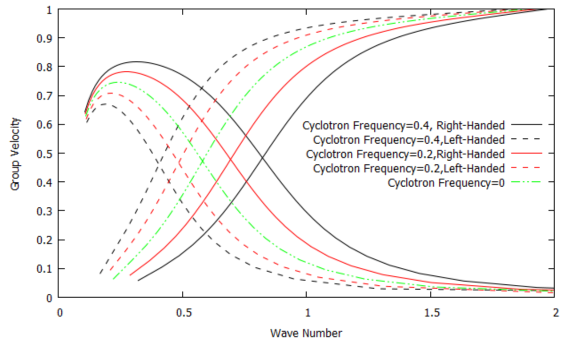

Fig. 5 Group velocity, $l = 0.03$ and $a_0 = 0.05$.

Now that we have the numeric data for ω and k, we may proceed to find the transverse profile modes of the laser from Eq. (11). Fig. (6) shows the transverse profile modes for ωs which has been labelled in Fig. (1), in two separate graphs for right- and left-hand polarizations. The behaviour of curves is different for different polarizations. We can see for right-hand modes, in a fixed $\bar{k}$ on the dispersion curves plane, width magnitude for modes is arranged by the increase in the group velocity. And the more group velocity the more width and the less localization. The inverse procedure is happening for left-hand modes.

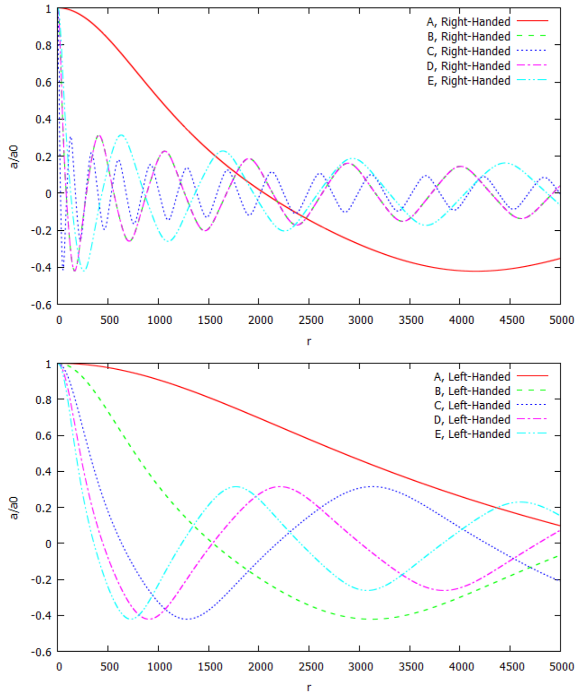

Fig. 6 Amplitude profiles $a_0 = 0.05$ and $l = 0.03$.

4 Conclusions

Nonlinear dispersion relation, group velocity and transverse amplitude profile are obtained for interaction of laser with magnetized nanoparticles lattice. It is shown that increase in the maximum amplitude and also in the ratio of nanoparticles radius to the separation lead to the localization of laser beam transversely. Furthermore, it is found that for the points with lower group velocities, left-hand polarized laser beam localizes more than points with greater group velocities.

References

[1] A. Chakhmachi and B. Maraghechi, Phys. Plasmas **18**, 022102 (2011).

[2] A. Ahmad and V. K. Tripathi, Nanotechnology **18**, 315702 (2007).

[3] J. R. Krenn, A. Dereux, J. C. Weeber, E. Bourillot, Y. Lacroute, and J. P. Goudonnet, G. Schider, W. Gotschy, A. Leitner, and F. R. Aussenegg, C. Girard, Phys. Rev. Lett. **89**, 2590 (1999).

[4] D. Y. Lei, A. Aubry, S. A. Maier and J. B. Pendry,New J. Phys. **12**, 093030 (2010).

[5] S. A. Akhmanov, A. P. Sukhurov, and R. V. Khokhlov, Sov. Phys.Usp. **10**, 609 (1968).