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کانونی شدن پالس شدید لیزر قطبیده‌ی خطی در اندرکنش با شبکه‌ای تناوبی از نانوذرات

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چکیده – خودهمگرایی یک پالس شدید لیزر منتشره در شبکه‌ای از نانوذرات مطالعه شده است. با استفاده از یک روش اختلالی معادله ی موج توصیف کننده اندرکنش غیر خطی لیزر با نانوذرات حاصل شده است. تحولات لکه لیزر با نمایه گوسی برای قطبش خطی بررسی شده است. ملاحظه شده است که نسبت شعاع ذرات به فاصله‌ی آنها دارای نقش اساسی در خاصیت خودهمگرایی می‌باشد. با پارامترهای فیزیکی قابل حصول خودهمگرایی می‌تواند از ناحیه‌ی طیفی مادون قرمز نزدیک تا ماوراء بنفش اتفاق بیافتد.

کلید واژه- اندرکنش لیزر-پلاسما، معادله ی غیر خطی شرودینگر، ناپایداری مدولاسیون

Intense Linearly-polarized Laser Pulse Focusing via Interaction with Periodic Lattice of Nanoparticles

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Abstract-The self-focusing of an intense laser beam propagating through a periodic lattice of nanoparticles is studied. Using a perturbative method, wave equation describing the nonlinear interaction of intense laser beams with nanoparticles is derived. The evolution of laser spot size with Gaussian profile for the linear polarizations is considered. It is found that the ratio of nanoparticles radius to separation have a crucial role in the self-focusing property. With available physical parameters, self-focusing can be observed from near IR to UV spectrum ranges.

Keywords: laser, nanoparticles lattice, nonlinear wave equation, self-focusing, spot size

Intense Linearly-Polarized Laser Pulse Focusing via Interaction with Periodic Lattice of Nanoparticles

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1 Introduction

Investigation related to the nanoparticles synthesis and their properties is one of the important recent interests with many applications in different scientific areas. Nonlinear interaction of laser with one-dimensional periodically nanostructured metals can lead to the localization or focusing of light in a region with characteristic dimensions smaller than the incident wavelength. This phenomenon called nano-focusing. Nanostructuring of metals can be obtained by the using of nanosized slits containing nonlinear materials [1], creating nanoholes on the metallic films [2] or nano-grating of metallic films [3]. Furthermore, such a nano-focusing property is observed theoretically and experimentally in the interaction of laser with determinate number or chain of nanoparticles [4]. A well-known nonlinear optical phenomenon that appears via the change of the medium refractive index exposed to the intense electromagnetic (EM) radiation is Self-Focusing (SF) [5]. During interaction of intense laser with medium, its refractive index increases by electric field of laser and it acts as a positive focusing lens for EM wave characterized by an initial transverse intensity profile. In all of the mentioned investigations about the nano-focusing, the aim of studies is the calculation or observation of intense localized EM energy at the near field zone in the vicinity of nanostructures. They have concentrated on the calculation of nanoparticles radiation via excitation of plasmons by microscopic electrodynamics. In this paper we are studying the problem of laser SF in the interaction with a bulk medium consisting of spherical nanoparticles.

2 Deriving Nonlinear Wave Equation

Let us consider the propagation of intense EM wave through periodic array of nanoparticles with radius r_c, separation d, and density of particles $N_c = 1/d^3$. We suppose that ions are immobile under interaction with high frequency EM wave and only spherical electronic clouds of each nanoparticle can respond to this frequency. In the equilibrium state, the electron density of the electronic cloud is constant n_0. Now, we suggest that the electric field of the laser beam is as below

$$\mathbf{E}_L = \hat{E}\hat{\mathbf{x}}\cos(kz-\omega t), \quad (1)$$

where, $\hat{E},\omega,k$ are the slowly varying amplitude, frequency and wave number of the laser, respectively. From Faraday's equation, magnetic field of the laser can be achieved as

$$\mathbf{B}_L = \frac{kc}{\omega}\hat{E}\hat{\mathbf{y}}\cos(kz-\omega t), \quad (2)$$

where c is the speed of light. Equation describing the relativistic interaction of laser fields with electronic cloud of can be written as

$$\begin{aligned}\frac{d}{dt}(\gamma\mathbf{v}) + \frac{\omega_p^2}{3}\mathbf{r} = -\frac{e}{m}\{\mathbf{E}(0) + (\mathbf{r}.\nabla)\mathbf{E}(0)\\ + \frac{1}{c}\mathbf{v}\times[\mathbf{B}(0) + (\mathbf{r}.\nabla)\mathbf{B}(0)]\},\end{aligned} \quad (3)$$

where γ, $\mathbf{v}$, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, $\mathbf{r}$, e and m, are relativistic Lorentz factor, velocity, plasma frequency of electrons, displacement of electronic cloud from equilibrium, magnitude of electron charge and electron rest mass, respectively and $\mathbf{E}(0)$, $\mathbf{B}(0)$ are the electromagnetic fields values in the centre of particles. Now we use the well-known perturbative method to solve Eq. (3). First order momentum equations is as following

$$\frac{dv_x^{(1)}}{dt} + \frac{\omega_p^2}{3}x^{(1)} = -\frac{e}{m}\hat{E}\cos(kz-\omega t), \quad (4)$$

where superscripts (1) refer to the first order perturbation. The solutions of Eqs. (4) is

$$x^{(1)} = \frac{ac\omega}{\omega^2 - \omega_p^2/3}\cos(kz-\omega t), \quad (5)$$

where $a = e\hat{E}/mc\omega$. The second order equation can be written as

$$\frac{d\mathbf{v}^{(2)}}{dt} + \frac{\omega_p^{\ 2}}{3}\mathbf{r}^{(2)} = -\frac{e}{mc}\mathbf{v}^{(1)} \times \mathbf{B}^{(1)}(0). \qquad (6)$$

By solving Eq. (6) one can find

$$z^{(2)} = \frac{a^2c^2k\omega^2 \sin(2kz - 2\omega t)}{2(\omega^2 - \omega_p^2/3)(4\omega^2 - \omega_p^2/3)} \ . \qquad (7)$$

Macroscopically, such a velocity can produce longitudinal modulation of electron density. For derivation of the electron density we use the following continuity equation

$$\frac{4\pi l}{3}\left(\frac{\partial n^{(2)}}{\partial t} + n_0 \nabla.\mathbf{v}^{(2)}\right) = 0, \qquad (8)$$

where $l = (r_c/d)^3$. By suggesting $n^{(2)} = \tilde{n}\cos(2kz - 2\omega t)$, we can derive

$$n^{(2)} = -\frac{a^2k^2c^2\omega^2 n_0 \cos(2kz - 2\omega t)}{(\omega^2 - \omega_p^2/3)(4\omega^2 - \omega_p^2/3)}. \qquad (9)$$

Third order equation can be obtained as

$$\frac{d}{dt}[\mathbf{v}^{(3)} + \gamma^{(2)}\mathbf{v}^{(1)}] + \frac{\omega_p^{\ 2}}{3}\mathbf{r}^{(3)} = -\frac{e}{m}\left[\frac{\mathbf{v}^{(2)} \times \mathbf{B}^{(1)}(0)}{c} + (\mathbf{r}^{(2)}.\nabla)\mathbf{E}^{(1)}(0)\right] \qquad (10)$$

where $\gamma^{(2)} = (1/2)(v^{(1)}/c)^2$. Equation (10) results

$$x^{(3)} = \frac{3a^3c\omega^3\cos(3kz - 3\omega t)}{4(\omega^2 - \omega_p^2/3)(9\omega^2 - \omega_p^2/3)}\left[\frac{k^2c^2}{(4\omega^2 - \omega_p^2/3)} + \frac{\omega^4/2}{(\omega^2 - \omega_p^2/3)^2}\right] + \frac{a^3c\omega^3\cos(kz - \omega t)}{4(\omega^2 - \omega_p^2/3)^2}\left[\frac{k^2c^2}{(4\omega^2 - \omega_p^2/3)} - \frac{3}{2}\frac{\omega^4}{(\omega^2 - \omega_p^2/3)^2}\right] \qquad (11)$$

After calculation of the electronic cloud velocity, we can derive the nonlinear macroscopic current density and substitute it in the following wave equation obtained from the Maxwell equations

$$\nabla^2\mathbf{E} - \nabla(\nabla.\mathbf{E}) = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}, \qquad (12)$$

here $\mathbf{J} = -(4\pi e l/3)(n_0 + n^{(2)})(\mathbf{v}^{(1)} + \mathbf{v}^{(2)} + \mathbf{v}^{(3)})$ is the macroscopic current density. Choosing only the fundamental harmonic of this current density and saving only the third order of dimensionless amplitude a, wave equation (12) reduces to

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{a} = k_p^2\left[\frac{l\omega^2}{(\omega^2 - \omega_p^2/3)} - a^2N\right]\mathbf{a}, \qquad (13)$$

here $k_p = \sqrt{4\pi/3}\,\omega_p/c$, $\mathbf{a} = \hat{\mathbf{x}}a\cos(kz - \omega t)$ and

$$N = \frac{3l\omega^4/4}{(\omega^2 - \omega_p^2/3)^2}\left[\frac{\omega^4/2}{(\omega^2 - \omega_p^2/3)^2} - \frac{k^2c^2}{(4\omega^2 - \omega_p^2/3)}\right].$$

3 Envelope evolution

To study the problem of SF we use the well-known source dependent expansion method [6]. In this method we expand the laser amplitude as a series of Laguerre-Gaussian source-dependent modes

$$a = \sum_m \hat{a}_m L_m(\chi)\exp[(i\alpha_s - 1)\chi/2], \qquad (14)$$

where $\chi = 2r^2/r_s^{\ 2}$, $r_s(z)$ is the spot size, $\alpha_s(z) = kr_s^{\ 2}/2R_c$ is related to the curvature R_c associated with the wave-front and $L_m(\chi)$ is a Laguerre polynomial of order m. By substituting Eq. (14) into wave equations (13) we can obtain

$$\frac{\partial^2 r_s}{\partial z^2} = \frac{4}{k^2 r_s^{\ 3}}\left(1 - k_p^{\ 2}a_0^{\ 2}r_0^{\ 2}N/8\right), \qquad (15)$$

where $r_s(z=0) = r_0$. The first term on the right-hand side of Eq. (15) gives the vacuum diffraction and the second term related to the effect of nonlinear current density of electronic clouds of nanoparticles on the laser spot size evolution. The parameter $P/P_c = k_p^{\ 2}a_0^{\ 2}r_0^{\ 2}N/8$ is the normalized power. When $P > P_c$ the nonlinear term is dominant and the SF occurs. The term $p_c = 2\pi^2c^5m^2/k_p^2\lambda^2e^2N$ (where λ is the wave-length of laser) presents the critical power for nonlinear SF of EM waves in the periodic lattice of nanoparticles. Solution of Eq. (15) is

$$\frac{r_s^{\ 2}}{r_0^{\ 2}} = 1 + \left(1 - \frac{P}{P_c}\right)\frac{z^2}{Z_R^{\ 2}}, \qquad (16)$$

where $Z_R = kr_0^{\ 2}/2$ is the Rayleigh length.

4 Numerical discussions

Let us consider nanoparticles $r_c = 40nm$. In the case of $d = 2r_c, 3r_c, 4r_c$, values of l are 0.125, 0.037 and 0.016, respectively. For all cases we set $r_0 = 15\mu m$. Figure 1 shows variation of normalized laser spot size r_s/r_0 with respect to normalized propagation distance z/Z_R when

$l = 0.02$, $(P/P_{cr}) = 1.52$, $\omega/\omega_{peff.} = 1.4$, $I_l = 2.4\times10^{15} W/cm^2$. Here laser power exceeds the critical power and the self-focusing of beam occurs. We can see that spot size reduces to zero at $Z_{sf} \approx 1.45 Z_R$. This is the effective distance in which catastrophic focusing starts. In table 1 we

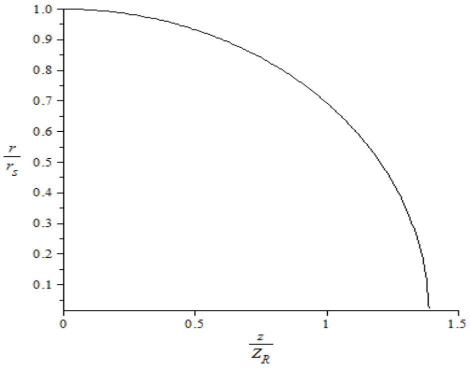

Fig. 1 Variation of spot size vs. propagation distance

Table 1 Physical parameters which match with dimensionless parameters of Fig. 1.

n_0 $(\times10^{22} cm^{-3})$	$\lambda(nm)$	a_0	$r_{\max}$ (nm)	Z_{sf} (mm)
0.17	1001.1	0.0420	20	0.92
1	412.8	0.0173	3.5	2.13
5	184.6	0.0077	0.7	4.78
15	106.6	0.0045	0.3	8.26

demonstrate a group of physical parameters that they match with the dimensionless parameters of Fig. 1. We can determine the laser frequency ω or wavelength λ by fixing the electron density n_0. For all cases we choose laser amplitudes so that it could produce power of $P = 6.29 GW$. In this table, $r_{\max}$ denotes the maximum amplitude of electronic cloud displacement and in our model it must be smaller than the nanoparticles radius r_c. Variation of n_0 from $0.17\times10^{22} cm^{-3}$ to $15\times10^{22} cm^{-3}$ causes the variation of wavelength λ from $1001.1 nm$ to $106.6 nm$ (from near IR to UV). The typical electron density magnitude for the Carbon, Silver and Aluminum are approximately $10^{22} cm^{-3}$, $5\times10^{22} cm^{-3}$, $15\times10^{22} cm^{-3}$, respectively. The SF distance associated to the catastrophic focusing regime is shown in the last column. In such a regime laser beam extremely focuses around the propagation axis and spot size r_s approaches to zero. This length varies from 1mm to 8mm. It should be better to mention that the wavelength $\lambda \approx 1\mu m$ (in the first row of this table) is used for the experimental investigations of laser-plasma interaction. Theoretical observations [7] show that the SF for practical cases of this area can occur in laser power and intensity roughly two orders of magnitude more than in our nanoparticles lattice.

5 Conclusions

Spot size evolution of linearly polarized laser pulse is studied. It is observed that the increase in the electronic cloud density and ratio of nanoparticles radius to their separation leads to the more focusing of laser. It is found that the SF by nanoparticles can occur from near IR to UV spectrum.

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