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مدلسازی تارهای بلور فوتونی نازک شده دارای طول موج پاشندگی صفر کاهشی

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چکیده - در این مقاله، برای اولین بار تارهای بلور فوتونی (*PCF*) نازک شده را که دارای طول موج با پاشندگی صفر (*ZDW*)کاهشی بر حسب طول خود هستند، از طریق حل معادله ویژه مقداری مد هدایت شده به روش تفاضل محدود در حوزه فرکانسی (*FDFD*) شبیه سازی می کنیم. از آنجا که پارامترهای ساختاری *PCF* های نازک شده، از قبیل قطر حفره های هوا *d* و گام شبکه *Λ*، در طول تار تغییر می کنند، لذا پاشندگی نیز در طول تار تغییر می کند. بنابراین دانستن رفتار دقیق پارامترهای پاشندگی برحسب طول *z*، برای کاربردهای اپتیک غیر خطی مورد نیاز است. در این مقاله، وابستگی طولی *ZDW* را برای مقادیر مختلف *d* و Λ شبیه سازی و روابط تحلیلی مفیدی برای توصیف ویژگی‌های مذکور پیشنهاد می‌کنیم. نتایج این مقاله که در توافق بسیار خوب با نتایج تجربی موجود است، برای کاربردهایی چون شبیه سازی انتشار پالس و همچنین تولید ابر پیوستار و تقویت پارامتری در *PCF* های نازک شده اهمیت دارد.

کلید واژه- تارهای بلور فوتونی، تفاضل محدود در حوزه فرکانسی، نازک سازی تار، پاشندگی.

Modelling of zero-dispersion wavelength decreasing tapered photonic crystal fibers

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Abstract- *In this paper, we investigate for the first time the dispersion properties of photonic crystal fibers (PCFs) with a continuously-decreasing zero-dispersion wavelength (ZDW) along their length, via solving the eigen-value equation of the guided mode using the finite-difference frequency domain (FDFD) method. Since the structural parameters such as air-hole diameter d and the pitch Λ of the microstructured cladding change along the tapered PCFs, dispersion properties change with the fiber length as well. Therefore, it is important to know the exact behaviour of such characteristics along z which is necessary for nonlinear optics applications. We simulate the z-dependency of the ZDW along the tapered PCFs for various d and Λ and propose useful analytical relations for describing such characteristics. The results of this paper which are in a very good agreement with the available experimental ones, are important for simulating pulse propagation as well as investigating nonlinear effects such as supercontinuum generation and parametric amplification in tapered PCFs.*

Keywords: *Photonic crystal fiber, finite-difference frequency domain, tapered fiber, dispersion.*

1 Introduction

Photonic crystal fibers (PCFs) are typically made of single material such as silica glass, with an array of microscopic air holes running along their length [1]. Solid core or index guiding PCFs which guide light by modified total internal reflection (TIR) have attracted great attention in recent years because of their unique properties that are not possible in conventional optical fibers [2].
A desirable property of PCFs is that, the additional design parameters of the air-hole diameter d and the hole-to-hole distance (pitch) Λ, offer much greater flexibility in the design to get desired dispersion and nonlinear characteristics [3].
A PCF can be tapered to achieve even smaller core than in an un-tapered PCF, and thereby reducing the required fiber length and the pump power for observation of efficient nonlinear effects such as supercontinuum generation [4]. In addition to the nonlinear optics, tapered PCFs have found interesting applications in sensing, refractometry, etc. [5-7]. Unlike the un-tapered PCFs where their parameters such as zero-dispersion wavelength (ZDW) are constant along the fiber (z-independent), these parameters are changed with z in tapered-PCFs [4]. Therefore, it is necessary to know the exact dependency of such parameters on z, when simulating the pulse propagation or investigating nonlinear phenomena in tapered PCFs [8].
In this paper, we solve the eigen-value equation for the tapered PCF [4] using the finite-difference frequency domain (FDFD) method to calculate the ZDW along the fiber and obtain the useful relations for describing its z-dependency. We also investigate the impact of air-hole diameter and pitch on such parameters. The obtained results which are in a very good agreement with experimental ones are important for accurate simulations of pulse propagation and nonlinear phenomena in the tapered PCFs where ZDW and the dispersion slope depend on length.

2 Theory and Simulation Method

If we start from the Maxwell equations for the electric field E and the Magnetic field H in a dielectric medium such as an optical fiber (in the absence of electric charge and current), after a few manipulations in the frequency domain we obtain the full-vectorial wave equations for the H_x and H_y components of the field which can be written as:

$$\nabla_\perp^2 H_x(x,y) + \varepsilon(x,y) k_0^2 H_x(x,y) + \frac{1}{\varepsilon(x,y)} \frac{\partial \varepsilon(x,y)}{\partial y} \left(\frac{\partial H_y(x,y)}{\partial x} - \frac{\partial H_x(x,y)}{\partial y} \right) = \beta^2 H_x(x,y) \tag{1}$$

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where H_x and H_y are x- and y- components of the magnetic field, respectively; and $k_0 = \frac{2\pi}{\lambda_0}$ is the free space wave number and λ_0 is the vacuum wavelength of the light. Also, $\varepsilon(x,y)$ is the permittivity of the dielectric medium and β is the propagation constant of the guide mode.
By applying the finite-difference approximation we can convert above equations into a numerical eigenvalue problem of the form [9]:

$$\phi H = \beta^2 H \tag{3}$$

where, ϕ is commonly referred to as the discretization matrix of the finite-difference problem which depends on the parameters such as permittivity, light wavelength, and step sizes of the grid denoted by Δx and Δy in the x- and y-directions, respectively. The complex propagation constant β can be solved out quickly and accurately in MATLAB using a sparse matrix solver [9, 10]. Therefore, we used the full-vectorial finite-difference method with the perfectly matched layer (PML) boundary conditions [11] to simulate dispersion characteristics of the tapered PCF. For a given geometry of the PCF with a fixed d and Λ, one can solve Eq. (3) and obtain the mode propagation constant β through which the effective index of the mode is given by $n_{eff} = \frac{\beta}{k_0}$. To obtain the dispersion curve of the PCF, one should repeat the calculations for a wide range of wavelengths to get the corresponding effective index for each wavelength. Finally, the dispersion is calculated using the relation as below [12]:

$$D = -\frac{\lambda}{c} \frac{\partial^2 \mathrm{Re}(n_{eff})}{\partial \lambda^2} \tag{4}$$

where c is the light speed and Re stands for the real part.

3 Results and Discussion

Fig. 1 shows the schematic illustration of a tapered PCF which is described by parameters as Λ_0, Λ_T and z_T, where $\Lambda_0 = \Lambda(0)$ is pitch of the un-tapered fiber, $\Lambda_T = \Lambda(z_T)$ is the pitch at the tapering waist, and z_T is the length from the start of the tapering to the tapering waist. It is assumed that the relative size d/Λ is constant along the tapering. This assumption seems reasonable if the PCF is tapered "fast-and-cold" [5]. To get the best fit to the experimental results presented in Ref. [4], we propose an analytical relation for the variation of the pitch Λ along the taper length z as:

$$\Lambda(z) = \left(\Lambda_0 - \Lambda_T\right)\left(\frac{\Lambda_T}{\Lambda_0}\right)^{\mu z/z_t} + \Lambda_T \qquad z \geq 0, \qquad (5)$$

where the border between un-tapered fiber and tapered fiber region is at z=0 and μ is the slope parameter which controls the slope of the tapering in Fig. 1. for Λ_0=4.8 μm, Λ_T=1.1 μm, and z_T=12m. The slope parameter is chosen as μ=5 to obtain the best fit to the experimental data.

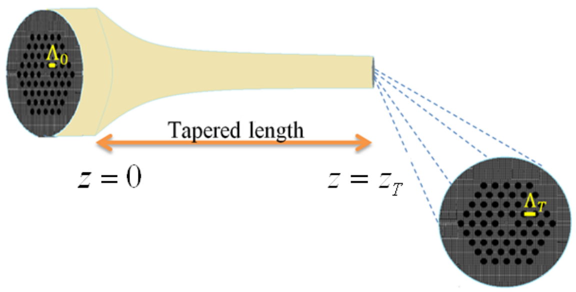

Fig.1. Schematic illustration of the tapered PCF with the pitch Λ_0 on the wider end and the pitch Λ_T on the narrower end. z_T is the length from the start of the tapering to the tapering waist.

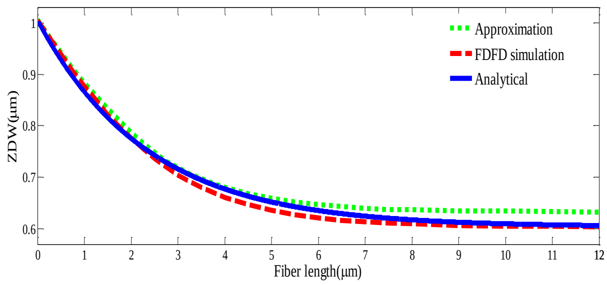

Fig.2. Z-dependence of the ZDW along the tapered PCF of Ref. [4] obtained via our FDFD simulation (red dashed line), and approximate method (green dotted line) where the PCF structure is simplified to a silica strand in air. The blue solid line corresponds to the analytical result proposed by Eq. (5). d/Λ is assumed to be 0.96 and other simulation parameters are Λ_0=4.8 μm, Λ_T=1.1 μm, z_T=12m, and μ=5.

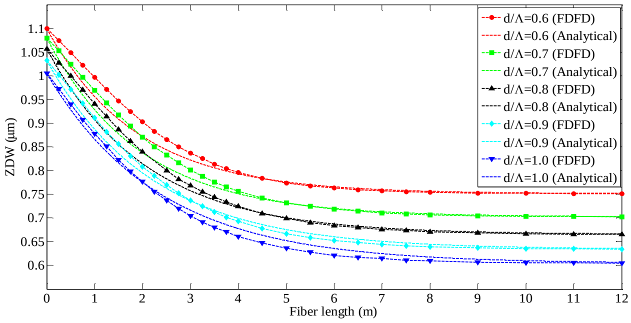

Fig.3. Zero-dispersion wavelength as a function of tapered PCF length for different hole-to-pitch ratios. The solid lines with and without symbols are related to the FDFD simulations and analytical formula (Eq. 6), respectively. Simulation parameters are the same as those in Fig. 2.

As it was discussed in Ref. [4] and is seen in Fig. 2, the variations of ZDW along the tapered PCF was obtained using silica strand model (green dotted line), since the ratio of the air-hole diameter to the pitch (d/Λ) was approximated by 1. This means that the microstructure of the PCF and the related air holes was ignored and the cladding was approximated by air. However, the exact solution which is based on the FDFD method can be different to that of the approximate solution as shown by the red dashed line in Fig. 2. Moreover, we propose the analytical relation for the best fit to the FDFD results which is given by:

$$ZDW(z) = \left(ZDW_0 - ZDW_T\right)\left(\frac{ZDW_T}{ZDW_0}\right)^{2\mu z/z_t} + ZDW_T \qquad (6)$$

where ZDW_0 and ZDW_T are the zero-dispersion wavelengths at the wider end (z=0) and at the narrower end ($z = z_T$) of the tapered PCF, respectively. Since d/Λ ~1, both the approximate and the exact (FDFD) results are very close to each other. In addition, the analytical results have a very good agreement to the FDFD ones which confirms the applicability of Eq. (6).

Naturally, for values of hole-to-pitch ratios less than 1, the ZDW behaviour versus the tapered length cannot be described by the approximate method and hence the FDFD method should be used. Fig. 3 shows ZDW variations along the tapered PCF length for different d/Λ using FDFD simulations (solid lines with symbols). As it is seen, the value of ZDW decreases as d/Λ increases and for large ratios when d/Λ =1, ZDW approaches the approximate value corresponding to the silica strand model (see Fig. 2). In addition to the FDFD results, we provide the analytical results (solid lines without symbols) obtained by the following formula:

$$ZDW(z) = \left(ZDW_0 - ZDW_T\right)\left(\frac{ZDW_T}{ZDW_0}\right)^{10\Lambda \, z/dz_t} + ZDW_T \quad (7)$$

In fact, Eq. (7) is a general form of Eq. (6), since in Eq. (6) we assumed that d/Λ ~1, therefore 2μ~10. As it is seen in Fig. 3, the analytical results match very well to the FDFD one especially for long z (longer than 4m). However, for shorter lengths the discrepancy is seen between both results which is less for higher values of d/Λ. In general, our model well describes the characteristics of ZDW decreasing tapered PCFs.

4 Conclusion

We have investigated the evolution of zero-dispersion wavelength (ZDW) along the tapered PCFs using the FDFD method. The results of our simulations were compared to those of experimental and a very good agreement was obtained. As the geometrical parameters (such as the air-hole diameter and the pitch) decrease along the tapered PCFs, the ZDW decreases as well. We also proposed analytical relations for the z-dependence of the pitch and ZDW which are useful for simulation and investigation of nonlinear phenomena in tapered PCFs.

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