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پایداری سالیتون‌های تاریک در مزدوج‌های موازی غیر خطی $\mathcal{PT}$-symmetric

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چکیده – در این مقاله، ابتدا با استفاده از روش اختلال به حل تحلیلی معادلات جفت شده در مزدوج‌های موازی غیر خطی *PT-Symmetric* **با پاشندگی مثبت که در یکی از موجبرهای آن بهره و در دیگری افت وجود دارد پرداخته‌ایم. جواب معمول غیر جفت شده این معادلات بصورت سالیتون تاریک است. محاسبه اختلال بصورت تحلیلی نشان میدهد که اختلالاتی که پایداری را تضمین می‌کنند به دو صورت روشن و تاریک هستند. این در حالی است که برای سیستمهای با جواب معمول سالیتون روشن صرفا یک اختلال سالیتون روشن پایداری آن را تضمین می‌کند.**

کلید واژه- اپتیک غیر خطی، سالیتون‌های تاریک، مزدوج‌های موازی غیر خطی PT-Symmetric

Stability of Dark Solitons In $\mathcal{PT}$-symmetric Nonlinear Couplers

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Abstract- In this paper we derive analytical dark soliton solutions in a nonlinear $\mathcal{PT}$-symmetric coupler with gain in one waveguide and loss in the other in normal dispersion regime by using perturbation method. The usual solutions of noncoupled equations are dark solitons. Evaluating perturbation analytically shows that perturbations which guaranty the stability are in two forms of bright and dark solitons. However, in systems with the usual solutions of bright solitons, only a bright soliton perturbation satisfies the stability.

Keywords: Nonlinear Optics, Dark Solitons, PT-Symmetric Couplers

1 Introduction

There has been a revolution in nonlinear physics over the past twenty years. Wave propagation models of physical media are naturally separated into two generic classes, conservative and dissipative. Recently, it was recognized that a more particular species of $\mathcal{PT}$ (parity-time)-symmetric systems may be identified at the boundary between these generic types which remain invariant under the combination of parity and time-reversal symmetry operation [1, 2]. They are represented by dissipative quantum mechanical models, and by classical waveguides. The concept has its roots in quantum mechanics where a $\mathcal{PT}$-symmetric non-Hermitian Hamiltonian may have an entirely real spectrum of eigenvalues [3].

The first experimental demonstrations of the $\mathcal{PT}$-symmetric effects in optics were in two waveguide directional linear couplers composed of waveguides with gain and loss [4]. Such systems are described by non-Hermitian Hamiltonians.

In quantum mechanics, the $\mathcal{PT}$-symmetric potential satisfies $v(x)=v^*(-x)$,*, denoting complex conjugation [5, 6].

In nonlinear optics, the $\mathcal{PT}$-symmetric potential is introduced by a complex refractive-index distribution $n(x)=n_R(x)+i\,n_L(x)$, which obey the condition $n(x)=n^*(-x)$, $n_R(x)$ stands for refractive index profile and the imaginary part, $n_L(x)$, represent the gain (+) or loss (-) in the system. Stable bright spatial solitons with PT-symmetric potentials have been reported recently [7].

However the stability of dark solitons in $\mathcal{PT}$-symmetric potentials in a nonlinear coupler with gain and loss is less studied.

In this paper, at first we obtain analytical solution of dynamical equations of dark solitons in nonlinear $\mathcal{PT}$-symmetric couplers and then discuss about the stability of these solutions.

2 Theory

Linear and nonlinear $\mathcal{PT}$-symmetric coupler with gain and loss in its two coupled waveguides has been studied theoretically and experimentally [3, 4, and 8].

In general form models which describes the spatial ore temporal propagation of light in planar optical waveguide and fiber coupler, are based on nonlinear Schrödinger (NLS) equations, coupled by the linear terms which indicate the tunnelling of light. To describe the temporal propagation of beams and pulses in $\mathcal{PT}$-symmetric nonlinear coupler the following two coupled dimensionless equations have been derived:

$$\begin{aligned} &iu_z \mp u_{\tau\tau} + 2|u|^2 u = -v + i\gamma u \\ &iv_z \mp v_{\tau\tau} + 2|v|^2 v = -u - i\gamma\, v. \end{aligned} \tag{1}$$

Here u and v are the normalized amplitudes variables in the top and bottom waveguides, z and τ indicate the length of fiber and normalized time, respectively. Plus and minus signs are stand for anomalous and normal dispersion respectively.

Two kinds of solitons have been discovered depending on dispersion sign. Bright solitons correspond to the positive sign and can be obtained by a solution in the form of $u(z,\tau)=A\sec h(z,\tau)\exp(iA^2 z)$ in anomalous regime. Similar to the case of bright solitons, dark solitons correspond to negative sign of dispersion. This means they occur in normal dispersive regimes.

They were discovered in 1973 and have attracted considerable attention since now [9-11]. The main difference compared with the case of bright solitons is that the domains of these solutions become a constant as $|\tau|\to\infty$.

To investigate the soliton solution in a $\mathcal{PT}$-symmetric nonlinear coupler a simple object composed a pair of nonlinearly coupled optical fibers is considered.

A schematic figure of a temporal $\mathcal{PT}$-symmetric is shown in figure (1).

To satisfy the $\mathcal{PT}$-symmetric condition we assume that the group velocities and second-order dispersion in waveguides are the same.

The coefficients in front of $u_{\tau\tau}$ and $v_{\tau\tau}$ are unity and Kerr nonlinearity coefficients in two wave guides have equal values, which is necessary for the existence of solitons [12].

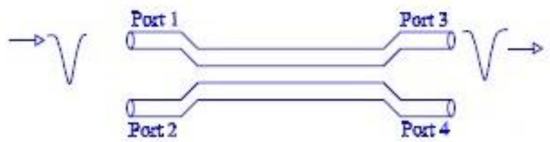

Figure 1: Scheme of nonlinear PT-symmetric coupler with gain in top waveguide and loss in bottom waveguide

The first term in right hand sides of equation (1) is related to coupling between the modes propagating in the two waveguides. The γ terms stand for the gain in one waveguide and loss in the other. Without loss of generality γ can be taken be positive, this means we have gain in the top and loss in the bottom waveguide. The gain and loss coefficients are taken equal to confirm the PT-symmetric condition [4].

2.1 Analytical solutions

In order to analyse Equation (1), it is convenient to represent the variables in the following form:

$$u(z,\tau) = exp(i(\Omega z - \theta))U(z,\tau), \quad v(z,\tau) = exp(i(\Omega z))V(z,\tau) \qquad (2)$$

Where θ is a constant angle, satisfy:

$$\sin\theta = \gamma \qquad (3)$$

And Ω is a real parameter which be chosen later. The substitution of Equation (2) into Equation (1) leads to:

$$iU_z - U_{\tau\tau} - \Omega U + 2|U|^2 U = -\cos\theta V + i\gamma(U - V)$$
$$iV_z - V_{\tau\tau} - \Omega V + 2|V|^2 V = -\cos\theta U + i\gamma(U - V). \qquad (4)$$

By applying $U = V \equiv \phi$ Equations (4) reduce into:

$$i\phi_z - \phi_{\tau\tau} - a^2\phi + 2|\phi|^2\phi = 0 \qquad (5)$$

Here $a^2 = \Omega - \cos\theta$.

By solving Equation (5) without loss of generality we reach into familiar dark soliton solution:

$$\phi(z,\tau) = a\tanh(a\tau)\exp(ia^2 z) \qquad (6)$$

We will be denoting, Equation (3), we will have $\cos\theta = \pm\sqrt{1-\gamma^2}$, and solitons exists only if $\gamma \prec 1$.

To solve coupled equations, we used perturbation method. In this method we take:

$$U(z,\tau) = \phi(\tau) + \delta U(z,\tau)$$
$$V(z,\tau) = \phi(\tau) + \delta V(z,\tau) \qquad (7)$$

and using symmetric and asymmetric combinations

$$p = \frac{\delta U + \delta V}{\sqrt{2}}, q = \frac{\delta U - \delta V}{\sqrt{2}} \qquad (8)$$

in linearized Equation (4) in δU and δV.

As we consider a separable solution for the linearized equation in the form of

$$p = \exp(\nu t)[(p_1' + ip_2')\cos\omega t + (p_1'' + ip_2'')\sin\omega t],$$
$$q = \exp(\nu t)[(q_1' + iq_2')\cos\omega t + (q_1'' + iq_2'')\sin\omega t]. \qquad (9)$$

In these equations we use

$p_1 = p_1' + ip_1''$, $p_2 = p_2' + ip_2''$

$q_1 = q_1' + iq_1''$, $q_2 = q_2' + iq_2''$

By introducing the operator

$$L = \begin{pmatrix} \frac{d^2}{d^2\tau} + \Omega - 6\phi^2 & 0 \\ 0 & \frac{d^2}{d^2\tau} + \Omega - 2\phi^2 \end{pmatrix}$$

We reach into two eigenvalue problems.

$$(L - \cos\theta)\vec{p} + 2\gamma J\vec{q} = \mu J\vec{p} \qquad (10)$$

$$(L + \cos\theta)\vec{q} = \mu J\vec{q} \qquad (11)$$

These equations have the matrix form as

$$\begin{pmatrix} L - \cos\theta & 2\gamma J \\ 0 & L + \cos\theta \end{pmatrix} \qquad (12)$$

The skew-symmetric matrix, "J" is

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The matrix in Equation (11) has two eigenvectors. One is $\begin{pmatrix} \vec{p} \\ 0 \end{pmatrix}$ and the second one is $\begin{pmatrix} \vec{p} \\ \vec{q} \end{pmatrix}$.

We have: $\vec{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \vec{q} = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$.

The first one is a linearized problem for unperturbed cubic Schrödinger equation and the spectrum of μ is stable. For the second eigenvector, Equation (10) becomes a nonhomogeneous equation and so it does not have any nonzero eigenvalues. Therefor the stability analysis reduces to solve Equation (11).

Defining $X = ax$, $\lambda = \mu / a^2$ and introducing $\eta = 2\frac{\cos\theta}{a^2}$, the eigenvalue problem (10) can be written as

$$\begin{pmatrix} L_1 + \eta & 0 \\ 0 & L_0 + \eta \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \lambda J \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \qquad (13)$$

Here $L_{0,1}$ are the scalar Sturm-Liouville operators

$$L_0 = d^2/dX^2 + 1 - 2\tanh^2 X$$
$$L_1 = d^2/dX^2 + 1 - 6\tanh^2 X \qquad (14)$$

The lowest eigenvalue of L_0 for the eigenfunction $\mathrm{sec}h(X)\tanh(X)$ is (-2) and for L_1 the lowest one, (-1), obtained for the eigenfunction $1 - \tanh^2 X$. As we can see for L_0 we reach into a bright soliton solution and for L_1 we obtain a dark soliton solution, which are the perturbed eigenfunctions and we can use them to guaranty the stability of soliton in this system. In this case we did not expect to have a look like bright soliton solution but in this method we have a perturbed eigenfunction which can help us to get into a condition for the stability of propgating a dark soliton in the $\mathcal{PT}$-symmetric coupler.

3 Conclusion

In this paper we obtain analytical solution of coupled NLS with nonlinearity in $\mathcal{PT}$-symmetric potentials in a coupler with gain in one waveguide and loss in the other one. The stability of these solutions is tested by analytical analysis with perturbation method. Evaluating perturbation analytically shows that perturbations which guaranty the stability are in two forms of bright and dark solitons. However the usual solutions of noncoupled equations are only dark solitons and for a bright solution, only a bright soliton perturbation satisfies the stability.

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