Designing an All Optical OR Logic Gate in a PT Symmetric Cell

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Abstract- An all optical OR logic gate based on a nonlinear optical parity time (PT) symmetric waveguide with triangular index profile is designed. In this design, we have taken the advantage of the non-reciprocity of PT symmetric systems as well as the asymmetric evolution of interacting solitons in such waveguides in nonlinear regime. The results demonstrate the potential applications of PT symmetric optical systems.

Keywords: Parity time symmetry, Nonlinear Schrodinger equation, Optical logic gate
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1. Introduction

The concept of parity-time (PT) symmetry—a property of physical systems that are invariant under time inversion and mirror reflection—was first introduced in the field theory of quantum mechanics [1-2]. In principle, in a PT-symmetric system the Hamiltonian operator ($\hat{H}$) satisfies the PT-symmetric conditions, namely $\hat{P}\hat{H}\hat{P} = \hat{H}$, wherein the parity operator ($\hat{P}$) acts as a spatial reflection (i.e., $\mathbf{r} \rightarrow -\mathbf{r}$) and the time reversal operator ($\hat{T}$) acts as a time reflection (i.e., $\mathbf{p} \rightarrow -\mathbf{p}$, $\mathbf{r} \rightarrow \mathbf{r}$ and $i \rightarrow -i$). From these, one can show that in a PT-symmetric system the real and imaginary parts of the potential $V(\mathbf{r})$ satisfy $\text{Re} V(\mathbf{r}) = \text{Re} V(-\mathbf{r})$ and $\text{Im} V(\mathbf{r}) = -\text{Im} V(-\mathbf{r})$, respectively. While the impact of PT symmetry in quantum mechanics is still open to debate, it has been recognized that one can flourish this concept into the realm of optics [3-4] and electronics [5] by deliberate use of a judicious balanced gain and loss. Specifically, the refractive index, $n(\mathbf{r})$, in the paraxial equation in optics, can play a similar role as potential, $V(\mathbf{r})$, does in the Schrödinger equation in quantum mechanics, due to the mathematical similarity. Researchers have paid a great deal of attention to PT-symmetric based optical systems, both theoretically [6-11] and experimentally [12,13], in recent years. One of the main components in all-optical signal processing techniques are all-optical logic gates. In recent years, the demands for all-optical signal processing techniques are rapidly increasing in telecommunication systems. Digital electronics are no longer capable of responding to the ever increasing future demands. In order to respond to such demands, many efforts have been performed. Different structures have so far been presented to recognize the performance of all-optical logic gates specifically those based on semiconductor optical amplifier (SOA) [14,15].

In this paper, we have aimed to design a simple all optical OR gate using the unusual features of PT-symmetric systems, similar to those of [10, 11]. When two fundamental bright spatial solitons are individually launched into two mirrored positions, in gain and loss regions, of a nonlinear PT symmetric waveguide their evolutions along the waveguide is shown to be nonreciprocal [9-11]. Moreover, when the two solitons are simultaneously launched into the gain and loss regions of a nonlinear PT symmetric waveguide, they interact in an asymmetric manner while propagating along the waveguide [11]. This is contrary to their symmetric interaction in a similar purely index guided profile [11]. Using these features we have designed a PT-symmetric all optical OR logic gate, for the first time.

2. Numerical Simulations

Assume a nonlinear medium of Kerr constant $n_2$ whose linear refractive index, $n_0$, is perturbed along the transverse $x$-direction by a complex and inhomogeneous profile, $\Delta n(x)$. The corresponding perturbing PT symmetric potential profile is $V(X) = k_0^2 w^2 \Delta n(X)$, wherein $X \equiv x/w$ is the dimensionless transverse coordinate and $k_0$ and $w$ are the free space wavenumber and the transverse dimension of the light beam, respectively [9]. The real and imaginary parts of the perturbing potential is assumed to be triangular and linear as in [10, 11]:

$$V(X) = \begin{cases} V_1 \left(1-|X|/b\right) + i V_2 |X|/b & |X| \leq b \\ 0 & |X| > b \end{cases} \quad (1)$$

wherein the dimensionless paramters $b$, $V_1$, and $V_2$ are half of the waveguide width and the maximum values of the real and imaginary parts of the profile, respectively. Using the Floquet-Bloch theory, we have already evaluated the threshold value of $V_{th} = 0.066$ for the PT symmetric phase breaking (phase transition) conditionin for linear regime in the 1D periodic array of the cell defined by Eq. (1) with $V_1 = 0.1$ and the periodicity of $2b=10$ [10, 11]. Note that the threshold for the nonlinear regime is always larger than that of the linear regime. Beyond the phase transition the eigenvalues for some Bloch wavenumbers become complex.
The paraxial propagation of a y-polarized optical wave of amplitude $E_y(x,z)$ along this nonlinear PT-symmetric waveguide can be described by the modified nonlinear Schrödinger equation (NLSE) [9, 11]:

$$i \frac{\partial \Psi}{\partial Z} + \frac{1}{2} \frac{\partial^2 \Psi}{\partial X^2} + V(X) \Psi + |\Psi|^2 \Psi = 0$$

(2)

where $\Psi(X,Z) \equiv k_0 w \sqrt{n_2/n_1} E_y(x,z)$ is the dimensionless amplitude of the electric field envelope and $Z \equiv z/k_0 w^2$ is the dimensionless longitudinal coordinate. In order to design the OR gate, we first assume the input field amplitude is the superposition of two fundamental bright solitons:

$$\Psi_{in} = \eta_1 \text{sech}(X - X_{01}) + \eta_2 \text{sech}(X - X_{02})$$

(3)

wherein the average transverse input positions $X_{01}$ (input 1) and $X_{02}$ (input 2) are assumed to be mirrored with respect to the origin (i.e., $X_{01} = -X_{02}$) and located in the gain and loss sides of the cell, respectively. Depending on the amplitudes of the input solitons $\eta_1$ and $\eta_2$, in Eq. (3), to be either zero or unity, there are four possible logic cases expected from the numerical solution of Eq. (2). Table 1 demonstrates these four possible states with the expected logic outputs.

As pointed out earlier in Section 1, the general asymmetric evolving behavior of these interacting solitons have already been demonstrated in [11]. Nevertheless, in this paper, by finding the right choices for the design parameters like the solitons average input positions and the waveguide length, we have made the PT-symmetric structures to act as a proper all-optical logic OR gate. For this purpose, the average transverse output position should fall in the gain side of the waveguide.

Top views of the intensity profiles of the evolving solitons along the PT-symmetric waveguide of Eq. (1) with $V_1 = 0.1$, $V_2 = 0.045$ and $b = 5$ for the logic states 2, 3, and 4 are illustrated in Fig. 1(a) with $X_{01} = -3.8$, Fig. 1(b) with $X_{02} = 3.8$ and Fig. 1(c) with $X_{01} = -X_{02} = -3.8$, respectively. For all three cases, shown in this figure, the waveguide lengths are the same ($Z = 250$), and the average output positions are all located within the gain side at about $X_{out} \approx -1.02$. Each profile is obtained by substituting Eq. (3) in (2) for the given initial input conditions and solving it by split step Fourier method (SSFM). The instantaneous peak intensity of each soliton in each case can be evaluated by the scale (color) bar depicted on left side of the corresponding figure. The output signal intensity for each case is also given in Table 1. Gate state 1 with zero inputs, obviously, is expected to result in no intensity profile and zero output. Comparison of Fig. 1(a) and 1(b) shows that the solitons dynamic for states 2 and 3 are non-reciprocal. This is due to the interplay of nonlinearity and the active elements of the PT-symmetric cell, the soliton is retarded

**Table 1.** The four possible logic states corresponding to the input solitons intensities $|\eta_1|^2$ and $|\eta_2|^2$ and related output intensities.

<table>
<thead>
<tr>
<th>State</th>
<th>Output</th>
<th>Input 1</th>
<th>Input 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-1.4</td>
<td>-1.4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
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<td>0</td>
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<td>-1.4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>-1.1</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

Fig. 1. Top views of the solitons intensity profiles along the PT-symmetric waveguide of length $Z=250$, representing the gate (a) state 2 with $X_{01} = -3.8$, (b) state 3 with $X_{02} = +3.8$, and (c) state 4 with $X_{01} = -X_{02} = -3.8$. 
while propagating along the gain side and is advanced while evolving along the loss side. On the other hand, as seen in Fig. 1(c), the dynamic behavior for two solitons along the PT-symmetric waveguide for the state 4 can be interpreted in the non-interacting and interacting regimes. In the non-interacting regime, where there is no overlap between the leading edge of the soliton launched into the waveguide at the gain side and the trailing edge of the one launched at the loss side of the waveguide, each soliton evolves as it has been launched individually, like those of states 2 and 3. In the interacting regime, where the aforementioned overlaps begin the retarded and advanced solitons interact incoherently (destructively). More details of the non-interacting and interacting dynamics for solitons propagating along triangular PT-symmetric cells can be found in [11].

As can be observed in Table 1, the output intensity of ~1.1 for the logic state 4 that is the consequence of incoherent (destructive) solitons interaction within the interacting regime is very close to the output intensity of ~1.4 for for both states 2 and 3 in the noninteracting regime. Possibility of obtaining such logic states that is a requirement for an OR gate action would not be possible via a similar waveguide with a purely real index guided profile.

3. Conclusion

Using SSFM, and taking the advantage of the interplay of nonlinearities and the active elements in a nonlinear triangular PT-symmetric waveguide we have demonstrated the possibility of designing an optical logic OR gate.

References