درهم‌تنیدگی و ناهم‌خوانی کوانتومی پایا برای کیوبیت‌های اتلافی توسط میدان مغناطیسی موضعی

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چکیده- سامانه‌ای شامل n کیوبیت غیربرهم‌کنشی در حال اتلاف در یک محیط مشترک را در نظر می‌گیریم. در حالی‌که سامانه در حالت پایه خود قرار دارد، یک میدان مغناطیسی جایگزیده برهم‌کنشی با یک کیوبیت، به منظور ایجاد درهم‌تنیدگی بین آن کیوبیت و کیوبیت دلخواه دیگر اضافه می‌نماییم. نتایج عددی به‌دست آمده نشان می دهند که خاموش‌کردن میدان مغناطیسی در زمان متناظر با بیشینه درهم‌تنیدگی (ناهم-خوانی)، منجر به ایجاد درهم‌تندیگی (ناهم‌خوانی) پایا برای سامانه‌هایی با تعداد متفاوت n می شود.

کلید واژه- اتلاف، درهم‌تنیدگی، کیوبیت، ناهم‌خوانی.

Stationary entanglement and discord for dissipating qubits by local magnetic field

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Abstract- We consider a system composed of n non-interacting qubits dissipating into a common environment. A local magnetic field interacting with a qubit is added to generate entanglement between the interacting qubit and each arbitrary qubit, while the system is in the ground state. Our numerical calculations show that turning the local magnetic field off at the maximum entanglement (discord) time, lead to a stationary entanglement (discord) for different system size n.

Keywords: Dissipation, Entanglement, Qubit, Discord.

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1 Introduction

Entanglement is a kind of quantum correlation which is known to be key resource of quantum communication and computation [1, 2] and has been verified in such protocols as cryptography [3] and teleportation [4]. Generation and manipulation of the entanglement in quantum systems are the necessary requirements for quantum information tasks, while coherent control of such systems to achieve entanglement is one of the main problems. However, the quantum systems would unavoidably interact with its environments by dissipative processes. Due to its fragility under environment-induced decoherence, entanglement can be observed only in the most elementary systems and on the shortest time scales. Nevertheless, achieving entangled states as stationary ones in open quantum systems has been interested with a variety of entanglement preserving mechanisms [5-9]. However, entanglement is not the only type of correlation useful for quantum technology. A different notion of measure, quantum discord, has also been proposed to characterize quantum correlation by Ollivier and Zurek [10] and independently by Henderson and Vedral [11]. Quantum discord is a measure of nonclassical correlations that may include entanglement but is an independent measure. This quantity has been investigated for some different spin systems [12]. We consider a system of an arbitrary number of non-interacting qubits dissipating into a common environment. Moreover, a local electromagnetic field interacting with one of the qubits is added within the quantum master equation [13]. Dynamics of the entanglement and discord between the interacting qubit and an arbitrary qubit have been computed as a function of time for different system size n. We found that the stationary entanglement (discord) can be obtained by turning the external magnetic field off at the maximum entanglement (discord) time. This amount of stationary entanglement (discord) is larger than the maximum amount of entanglement (discord) of the same non-interacting qubits [9].

2 SYSTEM DYNAMICS

We consider a system of n non-interacting qubits with associated Hilbert space $(C^2)^{\otimes n}$, dissipating into a common environment at zero temperature. Let $|0>$ ($|1>$) represents the ground (excited) state of a single qubit. Dynamics of this system will govern by the Lindblad master equation [13]:

$$\frac{\partial\rho}{\partial t}=\frac{\Gamma}{2}[2\sigma\rho(t)\sigma^{\dagger}-\sigma^{\dagger}\sigma\rho(t)-\rho(t)\sigma^{\dagger}\sigma], \qquad (1)$$

with $\sigma=\sum_{i=1}^{n}\sigma_i$ ($\sigma_i=|0><1|$) and Γ is the spontaneous decay rate. Amount of entanglement between each pair of qubits of this system depends on the number of initial excitation. The concurrence is a suitable measure of the degree of the entanglement for arbitrary bipartite mixed state [14],

$$\begin{aligned} &C=\max\{0,2\lambda_{\max}-tr\sqrt{R}\},\\ &R=\rho\sigma^{y}\otimes\sigma^{y}\rho^{*}\sigma^{y}\otimes\sigma^{y}, \end{aligned} \qquad (2)$$

where $\lambda_{\max}$ is the largest eigenvalues of the matrix $\sqrt{R}$. We assume a local magnetic field in place of the kth qubit with interaction term $H_s=-\vec{B}.\vec{\mu}$, where $\vec{\mu}=\gamma\hat{S}\otimes I^{n-1}$ is the magnetic dipole moment of the qubit k and γ is the geometric ratio [15]. Dynamics of this system in the Schrodinger picture will govern by the master equation

$$\frac{\partial\rho}{\partial t}=-i[H_s,\rho]+\frac{\Gamma}{2}[2\sigma\rho(t)\sigma^{\dagger}-\sigma^{\dagger}\sigma\rho(t)-\rho(t)\sigma^{\dagger}\sigma], \quad (3)$$

we assume that the system is in its ground state without any excitation ($\psi(0)=|00...0>$). In addition, small amount of magnetic field B = 0.1 with Γ = 1 are the best choices for these parameters [16]. The maximum amount of entanglement between interacting qubit (qubit k) and an arbitrary qubit has been obtained by setting the magnetic field along the x axis for n = 4. Therefore, we put the local external magnetic field along the x axis in the following calculation. The results are too cumbersome to express even for two qubits. Dynamics of the entanglement under a local magnetic field for n = 4 has been plotted in Fig. (1). Our numerical calculation shows that the maximum amount of entanglement can be preserved for the long time by turning the local magnetic field off at the corresponding time. Variation of the entanglement versus time for different system size n has been plotted in Fig. (2), while the magnetic field has been omitted at the maximum entanglement time.

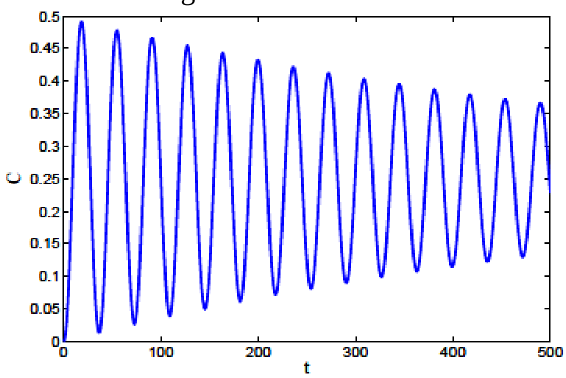

FIG. 1. Entanglement dynamics for n = 4 versus t (B = 0:1 and Γ =1).

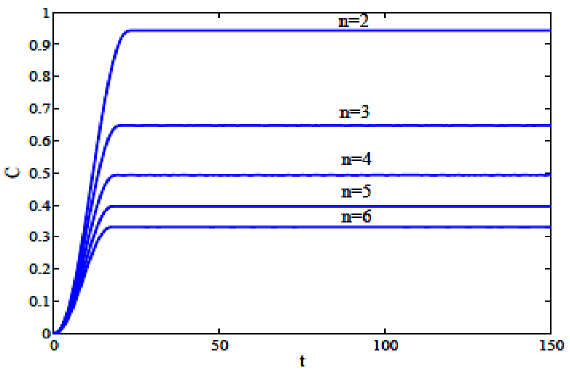

FIG. 2. Variation of the entanglement versus time while the magnetic field has been omitted at the maximum entanglement time. From top to bottom the system size is n = 2; 3; 4; 5; 6.

3 QUANTUM DISCORD

In classical information theory the mutual information which is the measure of the correlation between two arbitrary variable X and Y, reads [17]

$$I(X:Y)=H(X)+H(Y)-H(X,Y), \quad (4)$$

where $H(X)=-\sum_x P_{X=x}\log_2 P_{X=x}$ is the Shannon entropy while $P_{X=x}$ is the probability with X being x. Moreover, the joint entropy is $H(X,Y)=-\sum_x P_{X=x,Y=y}\log_2 P_{X=x,Y=y}$ which measures the total uncertainty of a pair of random variables X and Y. The mutual information can be rewritten into the equivalent expression

$$I(X:Y)=H(X)-H(X|Y), \quad (5)$$

where $H(X|Y)=-\sum_x P_{X=x|Y=y}\log_2 P_{X=x|Y=y}$ is the conditional entropy and $P_{X=x|Y=y}$ is the conditional probability of x being the realisation of the random variable X knowing that y is the realisation of the random variable Y . However, for the quantum systems the expressions of mutual information are not equivalent. In the quantum extension of above scenario, the classical probabilities are replaced by density operators, the summation is replaced by trace and the Shannon entropy is replaced by the von Neumann entropy [18]

$$S(\rho)=-tr\rho\log_2\rho. \quad (6)$$

The quantum version of mutual information can be written as

$$\mathrm{I}(X:Y)=S(\rho_X)+S(\rho_Y)-S(\rho_{XY}), \quad (7)$$

where $\rho_X(\rho_Y)$ is the reduced density matrix of the ρ_{XY} . In order to generalize the second expression of the classical mutual information to the quantum domain, we require a specification of the state of X given the state of Y. For that reason we focus on perfect measurements of Y defined by a set of one dimensional projectors Π_j^Y, corresponding to the outcome j. The state of X, after the measurement is given by

$$\rho_{X|\Pi_j^Y} = \Pi_j^Y \rho_{XY} \Pi_j^Y / P_j, \quad (8)$$

Where $P_j = tr_{XY}(\Pi_j^Y \rho_{XY})$. Then the quantum conditional entropy takes the form

$$S(X:Y) = S(\rho_X) - S_{\Pi_j}(\rho_{X|Y}), \quad (9)$$

where $S_{\Pi_j}(\rho_{X|Y}) = \sum_j P_j S(\rho_{X|\Pi_j^Y})$. Classical correlation between X and Y defines by maximizing S(X : Y) over all $\{\Pi_j\}$ as

$$C(X:Y) = \max_{\Pi_j} S(X:Y). \quad (10)$$

and finally the quantum discord measuring the quantum correlation between the two quantum sub-systems X and Y reads [10]

$$QD(X:Y) = I(X:Y) - C(X:Y). \quad (11)$$

Similar to the previous calculation of the entanglement, quantum discord between the interacting qubit with magnetic field and another arbitrary qubit of the system has been calculated. Our numerical result of the quantum discord dynamics versus time for the system size n = 4 has been shown in Fig. (3).

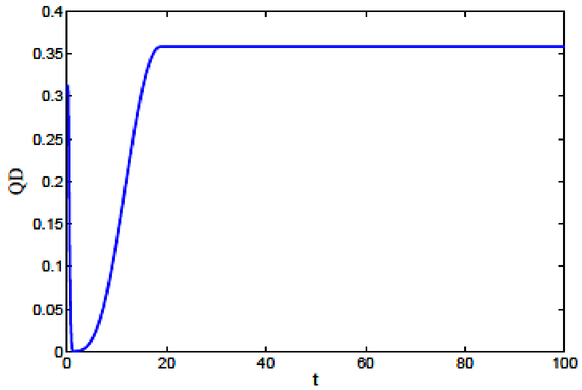

FIG. 3. (Color online)Quantum discord dynamics for n = 4 versus t while the magnetic field has been omitted at the time of the maximum amount of quantum discord.

4 CONCLUSION

We have presented a way to achieve stationary entanglement of the non-interacting qubits dissipating into a common environment. A local magnetic field is added to the system which interacts with one of the qubit leads to the entanglement between that qubit and other qubits. Due to the dissipating process, maximum amount of this entanglement decay as a function of time. We achieve a stationary entanglement (discord) by turning the local magnetic field off at the maximum entanglement (discord) time. This amount of stationary entanglement (discord) is larger than the corresponding amounts by just the dissipating process with the excited initial state.

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