The 27th Iranian Conference on Optics and Photonics (ICOP 2021), and the 13th Iranian Conference on Photonics Engineering and Technology (ICPET 2021).University of Sistan and Baluchestan, Zahedan, Iran, Feb. 2-4, 2021

کد مقاله : A-۱۰-۲۴۶۲-۱

رفتار زمانی یک اتم چهار ترازی با گذار چند فوتونی تحت چندین شرط خاص

بابک پروین

مراغه، دانشگاه مراغه، دانشکده علوم پایه، گروه فیزیک، صندوق پستی ۵۵۳-۵۵۱۳۶

چکیده- عملکرد یک اتم چهار ترازی با گذار چند فوتونی محصور شده در یک کاواک اپتیکی تک مد از لحاظ تئوری بررسی شده است. رفتار زمانی سامانه اتم-کاواک بر اساس معادله اصلی بیان شده است که در حل عددی آن، با استفاده از چندین کمیت فیزیکی، از روش رانگ-کوتای کلاسیکی مرتبه چهارم استفاده شده است. با اعمال چندین شرط خاص، اتم چهار ترازی رفتار یک اتم سه ترازی را به ازای هر گذاری از خود نشان می دهد که نتایج شبیه سازیها این فرآیند را تایید می کند.

کلید واژه-اتم چهار ترازی، روش رانگ-کوتای مرتبه چهارم، کاواک الکترودینامیک کوانتومی، گذار چند فوتونی، معادله اصلی.

Temporal Behaviour of a Four-Level Atom with Multiphoton Transition under Several Specific Conditions

Babak Parvin

Physics Department, Faculty of Basic Sciences, University of Maragheh, P.O. Box 55136-553parvin@maragheh.ac.ir

Abstract-The performance of a four-level atom with multiphoton transition confined in a single-mode optical cavity is investigated theoretically. The temporal behaviour of the atom-cavity system is delineated on the basis of the master equation, by using some physical quantities, the classical fourth-order Runge-Kutta method is utilized to solve it numerically. By applying multiple specific conditions, the four-level atom shows the behaviour of a three-level one for each transition, which the simulations results confirm this process.

Keywords: cavity quantum electrodynamics, four-level atom, fourth-order Runge-Kutta method, master equation, multiphoton transition.

1. Introduction

Here the time evolution of a four-level atom enclosed in an optical cavity is studied. A similar four-level atom has been investigated in [1], in which the transition between levels 2 and 3 occurs via one-photon, but here the multiphoton transition is considered between these two levels. In previous work, the behaviour of a three-level atom in a Λ configuration with multiphoton transition has been analysed in the steady state regime [2]. A closed set of equations describes the temporal behaviour of the atom-cavity system. To solve these equations numerically, the classical fourth-order Runge-Kutta approach is utilized with known initial conditions. By applying some circumstances along with a weak driving limit, the four-level atom illustrates the same behaviour as a three-level one for different transitions.

2. Four-Level Atom

The four-level atom with multiphoton transition is enclosed in the single-mode optical cavity as shown in Fig. 1. The transition from level 1 to level 4 is driven by a classical field with the Rabi frequency Ω at the resonance which is considered as a real quantity. The successive spontaneous emission rates from level 4 to level 1 are γ, Γ and ξ, respectively. A fully time description of the atom-cavity system is given by the following master equation [3,4]:

$$\begin{aligned}\dot{\rho} =& \\ &-\mathrm{i}\left[g\left(a^{q}\hat{A}_{32} + a^{\dagger q}\hat{A}_{23}\right) + \frac{\Omega}{2}\left(\hat{A}_{14} + \hat{A}_{41}\right), \rho \right] \\ &+(\gamma/2)\left(2\hat{A}_{34}\rho\hat{A}_{43} - \hat{A}_{44}\rho - \rho\hat{A}_{44}\right) \\ &+(\Gamma/2)\left(2\hat{A}_{23}\rho\hat{A}_{32} - \hat{A}_{33}\rho - \rho\hat{A}_{33}\right) \\ &+(\xi/2)\left(2\hat{A}_{12}\rho\hat{A}_{21} - \hat{A}_{22}\rho - \rho\hat{A}_{22}\right) \\ &+(\kappa/2)\left(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a\right),\end{aligned} \quad (1)$$

in which the second to the fourth terms refer to successive spontaneous emissions from atomic levels 4 to 1, respectively, and also the last term is related to the cavity decay with the rate κ. The coupling constant of the 2-3 atomic transition with the cavity field mode is g. The transition between the atomic levels 2 and 3 happens through the number of q photons at the resonance. The annihilation and creation operators of the cavity mode are a and $a^{\dagger}$, respectively. The atomic raising and lowering operators are $\hat{A}_{ij}$'s. Instead of solving directly the master equation (1), we use the following expectation values:

$$C_n = \left\langle a^{\dagger n} a^{n} \right\rangle, \quad (2)$$

$$H_n = \left\langle \hat{A}_{44} a^{\dagger n} a^{n} \right\rangle, \quad (3)$$

$$J_n = \left\langle \hat{A}_{33} a^{\dagger n} a^{n} \right\rangle, \quad (4)$$

$$F_n = \left\langle \hat{A}_{22} a^{\dagger n} a^{n} \right\rangle, \quad (5)$$

$$L_n = \mathrm{i}\left\langle \hat{A}_{41} a^{\dagger n} a^{n} - \hat{A}_{14} a^{\dagger n} a^{n} \right\rangle, \quad (6)$$

$$D_n = \mathrm{i}\left\langle \hat{A}_{32} a^{\dagger n-1} a^{n+q-1} - \hat{A}_{23} a^{\dagger n+q-1} a^{n-1} \right\rangle, \quad (7)$$

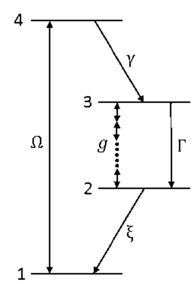

Fig. 1: The four-level atom with multiphoton transition which is trapped in the optical cavity.

equations (2) to (6) are established for $n \geq 0$, while the last equation holds for $n \geq 1$. The time evolution of the set of Eqs. (2) to (7) can be deduced from the following equations:

$$\dot{C}_n = g\sum_{m=1}^{q}\binom{q}{m}\left(n!/(n-m)!\right)D_{n-m+1} - \kappa n C \quad (8)$$

$$\dot{H}_n = -(\Omega/2)L_n - (\gamma + \kappa n)H_n, \quad (9)$$

$$\dot{J}_n = -gD_{n+1} + \gamma H_n - (\Gamma + \kappa n)J_n, \quad (10)$$

$$\begin{aligned}\dot{F}_n =& g\sum_{m=0}^{q}\binom{q}{m}\left(n!/(n-m)!\right)D_{n-m+1} + \Gamma J_n \\ &-(\xi + \kappa n)F_n,\end{aligned} \quad (11)$$

$$\dot{L}_n = 2\Omega H_n - \Omega C_n + \Omega F_n + \Omega J_n - \left(\left(\gamma/2\right)+\kappa n\right)L_n, \quad (12)$$

$$\dot{D}_n = -2gF_{n+q-1} + 2g\sum_{m=0}^{q}\binom{q}{m}\frac{(n+q-1)!}{(n+q-m-1)!}J_{n+q-m-1} - 0.5\left(\Gamma+\xi+\kappa\left(2n+q-2\right)\right)D_n. \quad (13)$$

The column vector ϕ_n is defined as follows:

$$\phi_n = \left(C_n \quad H_n \quad J_n \quad F_n \quad L_n \quad D_n\right)^T, \quad (14)$$

to determine the initial conditions, we assume that at the beginning of time the atom is in the ground state $|1\rangle$ and the field is in the coherent state $|\alpha\rangle$, in this case for $n \geq 0$, we will have:

$$\phi_n(0) = |\alpha|^{2n}\left(1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right)^T. \quad (15)$$

Now that the initial conditions are specified, the set of Eqs. (8) to (13) can be solved by the fourth-order Runge-Kutta method for any values of q [5].

3.Converting steps of the 4-Level Atom

The way of transforming the four-level atom confined in the cavity of Fig. 1 into the three-level one is discussed under some special conditions, in the three-photon transition case. We first assume that $\kappa \,\square\, 2g$, in this case, the transition from level 3 to level 2 takes place at the rate Γ_E:

$$\Gamma_E = 8g^2/\kappa, \quad (16)$$

in the next step, it is supposed to be $\Gamma \,\square\, \Gamma_E$. By applying these two circumstances, the four–level atom of Fig. 1 is turned into the four-level one in Fig. 2(a). In the weak driving limit in which $\Omega \,\square\, \gamma$, one can show that the population of the fourth atomic level is as follows:

$$A_{44} = \left(\Omega/\gamma\right)^2\left(1-\exp(-\gamma t/2)\right)^2, \quad (17)$$

since $\Omega \,\square\, \gamma$, so to a good approximation the fourth level can be omitted from the calculations, in thisoccasion, the transition from level 1 to level 3 occurs at the rate γ_E:

$$\gamma_E = \Omega^2/\gamma. \quad (18)$$

Thus, the four-level atom of Fig. 2(a) is converted to the three-level one of Fig. 2(b).By solving the related equations of the three-level atom, it can be shown that the time dependence of the third level population and the output mean photon number from the cavity, by applying the initial conditions, can be derived from the following equations:

$$P_3 = x_0\exp(-\lambda_- t) + y_0\exp(-\lambda_+ t) + z_0, \quad (19)$$

$$n = x_1\exp(-\lambda_- t) + x_2\exp(-\lambda_+ t) + x_3\exp(-\kappa t) + x_4, \quad (20)$$

in which we have:

$$\Delta = \gamma_E^2 + \Gamma_E^2 + \xi^2 - 2\gamma_E\Gamma_E - 2\xi\gamma_E - 2\xi\Gamma_E, \quad (21)$$

$$\lambda_+ = \left(\gamma_E + \Gamma_E + \xi + \sqrt{\Delta}\right)/2, \quad (22)$$

$$\lambda_- = \left(\gamma_E + \Gamma_E + \xi - \sqrt{\Delta}\right)/2, \quad (23)$$

$$z_0 = \xi\gamma_E / \left(\xi\left(\gamma_E + \Gamma_E\right) + \gamma_E\Gamma_E\right), \quad (24)$$

$$y_0\sqrt{\Delta} = \lambda_- z_0 - \gamma_E, \quad (25)$$

$$x_0\sqrt{\Delta} = -\lambda_+ z_0 + \gamma_E, \quad (26)$$

$$x_1 = 3\Gamma_E x_0 / \left(\kappa - \lambda_-\right), \quad (27)$$

$$x_2 = 3\Gamma_E y_0 / \left(\kappa - \lambda_+\right), \quad (28)$$

$$x_4 = 3\Gamma_E z_0 / \kappa, \quad (29)$$

$$x_3 = |\alpha|^2 - x_1 - x_2 - x_4. \quad (30)$$

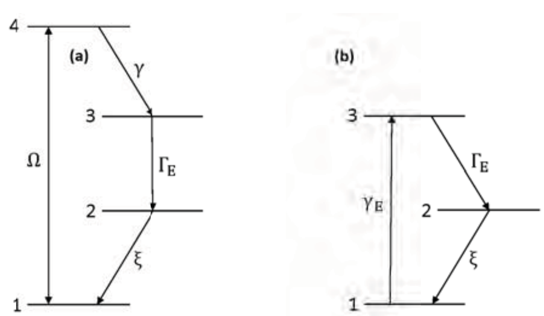

Fig. 2: The stages of converting the four-level atom of Fig. 1 to the effective three-level one.

In Figs. 3 and 4, the graphs of the third level population and mean photon number are depicted interms of the scaled time, in the dashed line based on the calculations of part 2 and in the solid line on the basis of the three-level atomic model for theone-photon and two-photon transitions.

Comparison of diagrams in these figures indicates that the four-level atom acts like the three-level one and the results are very close to each other.

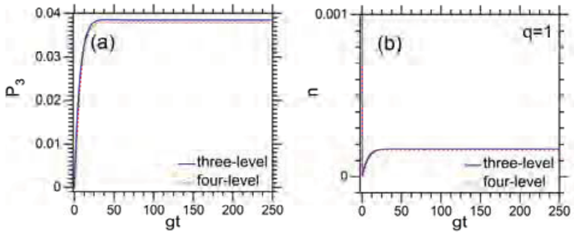

Fig. 3: The graphs of (a) third level population and (b) mean photon number versus the scaled time for $\kappa / g = 30$, $\Gamma / g = 0.003$, $\gamma / g = 3$, $\Omega / g = 0.13$, $\xi / g = 0.1$ and $|\alpha|^2 = 1$ in the one-photon-transition.

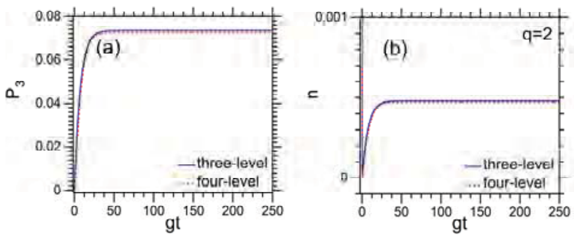

Fig. 4: The diagrams of (a) third level population and (b) mean photon number in terms of the scaled time for $\kappa / g = 35$, $\Gamma / g = 0.001$, $\gamma / g = 3$, $\Omega / g = 0.17$, $\xi / g = 0.15$ and $|\alpha|^2 = 1$ in the two-photon-transition.

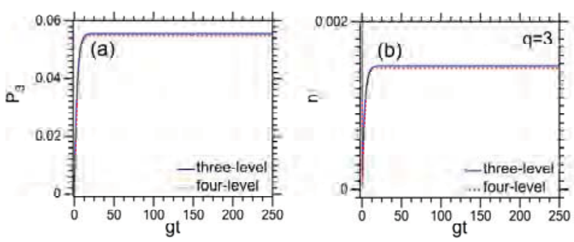

Fig. 5: The curves of (a) third level population and (b) mean photon number against the scaled time for $\kappa / g = 30$, $\Gamma / g = 0.003$, $\gamma / g = 4.1$, $\Omega / g = 0.26$, $\xi / g = 0.3$ and $|\alpha|^2 = 1$ in the three-photon-transition.

In Fig. 5, the third level population and mean photon number are drawn versus the scaled time, in the dashed diagrams based on the computations of part 2 and the solid curves in accordance with Eqs. (19) and (20) related to the three-level atom, in the three-photon transition case. By comparing the graphs in this figure, it can be said that under applied conditions, the four-level atom shows the same behaviour as the three-level one.

4. Conclusions

In this paper, the behaviour of the four-level atom trapped in the optical cavity has been investigated in the one-photon to three-photon transitions by applying several specific conditions, in the time-dependent regime. By using the closed set of physical quantities in order to solve the master equation, the fourth-order Runge-Kutta approach has been applied. In the three-level model, the related relationships of the third level population and the output mean photon number from the cavity have been obtained in the three-photon transition case. Eventually, the numerical simulations of the four-level atom have been compared to the acquired results of the related calculations of the three-level one for each transition. The depicted curves in Sec. 3 demonstrate that the four-level atom enclosed in the cavity indicates similar behaviours to the effective three-level one.

Acknowledgements

The author would like to thankDr. Howard J. Carmichael for his comment.

References

[1] Y. Mu and C.M. Savage, "One-atom lasers," Phys. Rev. A, Vol. 46, pp. 5944-54, 1992.
[2] B. Parvin, "The improvement of lasing without inversion in the presence of multi-photon transition in a three-level atom confined in an optical cavity in the steady-state regime," Eur. Phys.J. Plus, Vol. 132, pp. 180 (1-14), 2017.
[3] H.J. Carmichael, Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck Equations, Springer-Verlag, 1999.
[4] H.J. Carmichael, Statistical Methods in Quantum Optics 2: Non-Classical Fields, Springer-Verlag, 2008.
[5] G.E. Forsythe, M.A. Malcolm, and C.B. Moler, *Computer Methods for Mathematical Computations*, Prentice-Hall, 1977.