Abstract- We investigate the interaction of self-pulsing laser cavity solitons (SPLCS) in a broad-area semiconductor laser with saturable absorber. We show that for appropriately close distances, synchronization is excepted to happen between SPLCSs. As a result of such an interaction, the two elements drive each other away. Since SPLCSs are free in choosing their phase and polarization, phase interaction between two entities leads to a synchronized behavior in terms of phase and intensity oscillations. We show that interaction strength depends on the initial separation distance of SPLCSs and it is more effective in shorter ranges. We also show how the manifestations of such an interaction, like velocity of SPLCSs, decrease exponentially with the initial separation distance.

Keywords: cavity soliton, self pulsation, synchronization
1. Introduction

Synchronized behaviour has been studied in a vast variety of systems like synchronously flashing fireflies [1], pacemaker cells in mammalian hearts [2], and a network of microwave oscillators [3]. It has been confirmed that coupling among the constituents of a system plays a key role in their mutual synchronization. This concept has been extended to the optical systems, such as networks of coupled monochromatic lasers, leading to demonstration of phase-locking [4] and coherent beam combining [5], and cavity and spatial soliton synchronization [6,7].

Lasers are a prominent example for self-sustained nonlinear oscillators. Laser Cavity Solitons (LCSs) which form in a broad-area semiconductor laser with saturable absorber, defined as localized spots of light surrounded by lower intensity emission, make use of the additional freedom of optical phase in such free running lasers. Here we investigate self-pulsing Laser Cavity Solitons (SPLCSs) whose existence depends on certain instability provided by Andronov-Hopf bifurcation. Self-pulsing, also known as passive Q-switching, has its origin in the interplay of the slowly responding population differences in the amplifier and absorber media and the fast response of the electric field intensity in the cavity. In a cavity where the amplifying medium is excited to a sufficiently high level through some pumping process, the electric field intensity increases if the unsaturated gain overcomes the losses. After a while, the absorber saturates which leads to a greatly enhanced output power and subsequently to saturation of the gain, which in turn allows the field to decay to almost zero intensity. Then the process repeats itself giving rise to a pulse train with a typical frequency of the order of several gigahertz in semiconductors [8].

Since it is possible to have multiple LCSs in the transverse section of a broad-area semiconductor laser with saturable absorber, it is interesting to investigate the mutual interaction of two SPLCSs when they are switched in different locations. Synchronization is expected to happen due to the phase-unlocked nature of these localized optical elements.

2. Model

The basic equations describing a broad-area semiconductor laser with saturable absorber are [9]:

\[
\begin{align*}
\dot{F} &= \left[ (1-i\alpha)D + (1-i\beta)d - 1 + i\nabla^2 \right] F \quad (1) \\
\dot{D} &= b_1 \left[ \mu - D \left( 1 + F^2 \right) - BD^2 \right] \quad (2) \\
\dot{d} &= b_2 \left[ -\gamma \left( 1 + sF^2 \right) - Bd^2 \right] \quad (3)
\end{align*}
\]

where the dimensionless variables \( F, D \) and \( d \) are, respectively, the slowly varying envelope of the electric field, the carrier density of the amplifier and the carrier density of the absorber defined as \( d = \eta_1 N_2 / N_{02} - 1, \ D = \eta_2 N_1 / N_{01} - 1 \) where \( N_1 \) and \( N_2 \) are the carrier densities in the active and passive materials, respectively, \( N_{1,0} \) and \( N_{2,0} \) are their transparency values, \( \eta_1 \) and \( \eta_2 \) are a dimensional coefficients related to gain and absorption, respectively.

The parameters \( \alpha, \beta \) are the linewidth enhancement factors in the amplifier and in the absorber, respectively; \( \mu \) is the pump parameter, \( \gamma \) is the unsaturated absorption, \( s = a_1 b_1 / a_2 b_2 \) is the saturation parameter, i.e. the ratio of saturation intensity in the amplifier and in the absorber. \( b_1 \) is the ratio of photon lifetime to carrier lifetime \( \tau_p / \tau_w \) in the amplifier and \( b_2 \) is that in the absorber. Therefore, one can define \( r \) as the ratio of carrier lifetime in the amplifier and in the absorber.
Finally, the coefficient \( B \) is the rate of radiative recombination in semiconductor. Time is scaled to the photon lifetime, and space is scaled to the diffraction length. Typically, a time unit is a few picoseconds and a space unit is \( \sim 4 \, \mu m \).

3. Results and discussion

The study of the interaction of two SPLCSs in the semiconductor laser with saturable absorber showed that for distances greater than \( 42 \, \mu m \), they can stably exist beside each other. For a short time after creation of SPLCSs, their intensities oscillate in an opposite phase. However, as a result of interaction, they first synchronize in phase and intensity and then start moving away from each other. Figure 1 shows the intensities of the SPLCSs before and after synchronization.

![Figure 1: The intensity of SPLCSs before synchronization (a, b) showing opposite phase oscillations and after synchronization (c, d) which depicts in-phase intensity oscillation.](image)

Synchronization in the intensity variation of the two SPLCSs can be understood from the time trace of their intensity difference reaching zero after a certain time known as synchronization time. Figure 2 (left panel) shows the phase difference locked at \( \pi \) after synchronization.

![Figure 2: (Left) Intensity difference and (Right) phase difference of the two interacting SPLCSs for \( r = 1.7 \) and the initial distance of \( 48 \, \mu m \).](image)

The synchronization time of the SPLCSs depends on their initial separation distance and the bifurcation parameter \( r \). Figure 3 shows the synchronization time \( t_{\text{sync}} \) versus the initial separation distance \( d_{\text{in}} \). It is clearly seen that by switching SPLCSs in farther distances from each other, the time required for their synchronization increases with an exponential scale.

![Figure 3: Synchronization time versus the initial separation distance between SPLCSs for different values of the bifurcation parameter \( r \).](image)

On the other hand, we see from figure 4 that by increasing \( r \) the time required for synchronization decreases exponentially which is common for all values of initial separation distance between the two SPLCSs. As already mentioned, the synchronization of the two SPLCSs as localized self-sustained oscillators leads to a repulsive force which moves them away from each other. Their displacement velocity is very fast at the beginning and decreases as they get farther. The trend is again exponential which further evidences an...
exponential form for the interaction potential. As $r$ increases, the displacement velocity increases accordingly which is consistent with figures 3 and 4 where we showed that synchronization time is shorter for larger $r$ values resulting in a faster displacement. Figure 5 shows the displacement time for different values of $r$ and a fixed initial separation distance.

![Figure 4: Synchronization time for different initial distances versus the bifurcation parameter $r$.](image)

![Figure 5: The time needed for the SPLCSs to move away from each other in the amount shown on the horizontal axis for different values of the bifurcation parameter $r$. Their initial spacing is 48µm.](image)

### 4 Conclusion

In this paper, we showed that if two self-pulsing laser cavity solitons are switched at a proper distance from each other, the mutual interaction leads to synchronization and subsequently a repulsive behaviour will emerge. By increasing the initial separation distance between two such optical elements, the time needed for the synchronization to establish increases exponentially and, as a result, the velocity at which they repel each other decreases. Also, with increasing the bifurcation parameter representative of the ratio of carrier life times in the amplifier and absorber materials $r$, the synchronization time decreases and the velocity increases accordingly which is again exponential. All these exponential trends suggest that the interaction potential governing such repulsive behaviour is of exponential form. We believe that such a spatial adjustment of laser cavity solitons driven by a synchronized behaviour can be extended to clusters of self-sustained oscillators with promising collective and emergent properties.

### References


