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بررسی تحلیلی فرامواد هذلولی با لایه‌های فلزی بسیار نازک

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چکیده- در این مقاله یک مدل جدید برای طراحی عملکرد فرامواد هذلولی با لایه‌های فلزی بسیار نازک پیشنهاد شده است. براساس تحلیل تئوری، رابطه‌ای جدید برای مدل درود اصلاح‌شده در لایه‌های فلزی با ضخامت کمتر از ده نانومتر پیشنهاد شده است. سپس اثر این رابطه به گذردهی لایه‌های متعامد و موازی اعمال شده است شده است. سپس، منحنی‌های پاشندگی، انتقال و بازتاب برای این ساختارها اصلاح گردیده است. محاسبات عددی نشان داده است، وقتی اثر فلزات بسیار نازک را به ساختارهای HMM اعمال می‌کنیم، بخش‌های حقیقی و موهومی دامنه موثر گذردهی به‌ترتیب حدود ۴۵/۲۸٪ و ۵۲/۶۸٪ کاهش می‌یابد. این میزان تغییرات در دامنه گذردهی موثر ضریب انتقال را در حدود ۱۵٪ محدود می‌کند.

کلید واژه- فرامادە، ثابت میرایی، مدل درود اصلاح‌شده

Analytical investigation of hyperbolic metamaterial with ultra-thin layer

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Abstract- We present a new model for performance analysis of hyperbolic metamaterial with ultra-thin metal layer. Based on theoretical analysis, we have been modified Drude relation for metal layers with geometric size below ten nanometers. Then, the effect of this relation applied to transverse and perpendicular effective permittivity. Then, dispersion relation and transition and reflection relations coefficient for these structures are modified. The numerical calculation show, when we apply the effect of ultra-thin metal in HMM structure, the magnitude of effective permittivity is reduced about 45.28% in real part and 52.68% in imaginary part. These variations in effective permittivity limit the magnitude of transmission coefficients to 15%.

Keywords: Metamaterial, damping constant, modified Drude model

1. Introduction

Metamaterial research have attracted much attention because of their promising applications in imaging [1], cloak of invisibility [2], sensing [3], waveguide [4] and switching [5]. The hyperbolic metamaterial (HMM) are artificial nanostructures with extremely anisotropic, uniaxial and subwavelength metal/dielectric units. In metamaterial, the dispersion relation engineering is used in order to provide unique electromagnetic modes. In fact, HMM can be considered as polariton crystals that coupled light and material states to larger bulk electromagnetic states [6].

For isotropic media, linear and isotropic dispersion behaviour of propagation waves lead to spherical isofrequency surfaces, which is expressed by $k_x^2 + k_y^2 + k_z^2 = \varepsilon\, \omega^2/c^2$ [7]. In HMM, As derived from their name, isofrequency contour (k_x, k_y, k_z) in these structures is hyperbolic. It means, an HMM structure can be considered as a uniaxial anisotropic metacrystal with extremely anisotropic dielectric tensor $\varepsilon = [\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}]$, whrer $\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon_{\parallel}$ are longitudinal and $\varepsilon_{yy} = \varepsilon_{\perp}$ transverse components of permittivity. Thus, in these medium, we can express isofrequency relation as $\frac{k_x^2+k_z^2}{\varepsilon_\perp} + \frac{k_y^2}{\varepsilon_\parallel} = \frac{\omega^2}{c^2}$. However, when there is extreme anisotropy such that $\varepsilon_\parallel . \varepsilon_\perp < 0$, the isofrequency surface changes into hyperboloid. Such a phenomenon requires the material to behave like a metal in one direction and a dielectric in the other [8]. This can be achieved using artificial engineered nanostructured metamaterial.

Widespread works has been proposed on hyperbolic metamaterial in various applications. It can be argued that these works, damping constant (in Drude model) is usually considered to be a constant. The fixed value of damping constant, however, is no longer valid, when the geometric size of the metal goes below tens of nanometres. Actually, in HMM structures, the effect of ultra-thin metal -which has a significant effect on effective permittivity in both transverse and perpendicular directions- is not considered [9]. While, in practice, decrease the width of metal layers in HMM can change the overall system performance. In this paper, we proposed a new model to analytical investigation of ultra-thin metal. Then, we extracted the relations to investigate the effect of it on effective permittivity as well as dispersion, transmission and reflection relations. Finally, theoretical results are compared with simulation ones.

2. The proposed model for nanolayers metal

The proposed structure to design of HMM consists of periodic dielectric/metal layers. In these structures, we usually use modified Drude model as (1) to simulate the permittivity of metal,

(1) $$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega}$$

In (1), $\omega_p = \sqrt{\frac{ne^2}{\varepsilon_0 m^*}}$ is the volume plasma frequency, Γ is damping constant, ε_∞ is a sum or integral value after taking all pertinent transitions into account, which usually replace by a constant value. When we investigate the interaction of light with nanostructured metal, the characteristics of metal due to size effect will be modified. In the Drude free electron model, parameter Γ is usually replace with a constant at a given temperature. When the geometric size of metal goes below tens of nanometres, the fixed value of Γ is not valid. The damping constant Γ is defined as v_F/l. Γ is a collision rate related to an electron's mean free path l in the metal. When the length scale of the continuous metal portion of the metamaterial unit cell is comparable to or smaller than l, the movement of free electrons is further limited by the physical boundary of the metal structure, and the effective mean free path is reduced according to [10],

(2) $$\frac{1}{l^*} = \frac{1}{l} + \frac{1}{l_1}$$

where l_1 represents the size of the metal particle and l^* is the size-limited mean free path of electrons. By replacement (2) in the definition of Γ, we have

(3) $$\Gamma^* = \Gamma + \xi \frac{v_f}{l_1}$$

Where ξ is on the order of one and depends on the geometry specifies and some other factors. Therefore, in order to calculate the permittivity of nanostructured metals, we should be used as (3) in the modified Drude model. Now, we introduced a new modified Drude model for ultra-thin layers as,

(4) $$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \Gamma^{*2}} + i\frac{\omega_p^2 \Gamma^*}{\omega(\omega^2 + \Gamma^{*2})}$$

As it is clear from eq. (4), the real part of the dielectric function is only marginally related to $\boldsymbol{\Gamma}^*$ while the imaginary part is proportional to $\boldsymbol{\Gamma}^*$.

2.1. Effective medium theory for multilayers HMM

The proposed HMM structure composed of 15 vertically stacked unit cell that the HMM slab with unit cells made of dielectric/metal nanolayers. The unit cell, as illustrated in fig. 1, is composed of a 8nm thick with a dielectric constant of $\varepsilon_d = 4.5$, and 5nm thick nanolayers of gold with a dielectric constant of ε_m, taken from experimental data and based on (4).

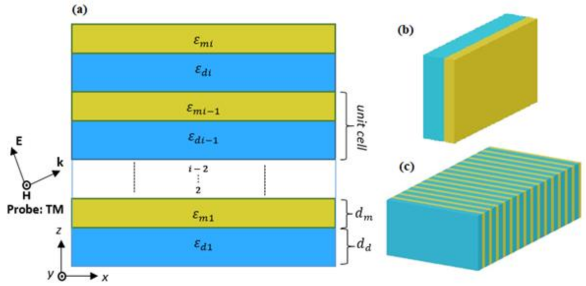

Fig. 1. (a) Side xz view of the HMM structure, under illumination of a TM polarized probe, (b) 3D view of the unit cell and (c) 3D view of the proposed structure. Each cell contains two layers of metal and dielectric with a thickness of 13nm. The entire structure of the switch consists of 15 cells with of 195nm thick.

Effective permittivity tensor $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ can be approximated using Maxwell equation and the effective medium theory (EMT) [9],

$$\varepsilon_{\parallel} = \sum_{i=1}^{m} f_i \varepsilon_i \tag{5}$$

and

$$\varepsilon_{\perp} = \left[\sum_{i=1}^{m} f_i \varepsilon_i^{-1}\right]^{-1} \tag{6}$$

With $i = 1, 2, 3, \ldots, m$ and m represents the total number of layered in HMM. f is filling factor and defined as,

$$f_i = \frac{d_i}{\sum_{i=1}^{m} d_i} \tag{7}$$

2.2. Dispersion relation, Transmission and Reflection coefficients

The dispersion equation for TM-waves in the infinite periodic structure, which relates the frequency ω and the Bloch wave number k^{TM}, can be written as [11],

$$k_z^{TM} = \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_{\parallel} - \frac{\varepsilon_{\parallel}}{\varepsilon_{\perp}} k_x^2}. \tag{8}$$

the equations for reflection and transmission coefficients for the system derived as,

$$r = \frac{-i\frac{1}{2}\left(\varsigma_{TM} - \frac{1}{\varsigma_{TM}}\right)\sin\left(k_z^{TM} L\right)}{\cos\left(k_z^{TM} l\right) - i\frac{1}{2}\left(\varsigma_{TM} + \frac{1}{\varsigma_{TM}}\right)\sin\left(k_z^{TM} L\right)}$$
$$t = \frac{1}{\cos\left(k_z^{TM} L\right) - i\frac{1}{2}\left(\varsigma_{TM} + \frac{1}{\varsigma_{TM}}\right)\sin\left(k_z^{TM} L\right)} e^{ik_0 L\cos\theta_i} \tag{9}$$

Where, L is the length of HMM and $\varsigma_{TM} = \varepsilon_{\perp}\frac{\omega}{c}\cos\theta_i / k_z^{TM}$.

3. Simulation Results

The theoretical results presented in this paper are confirmed by several computer numerical simulations for a system shown in fig. 1. First, by using (4), the effect of thickness is shown for various width of metal layer. Based on (4) the real and imaginary parts of dielectric constant for bulk, 50, 10, 8, 5 and 3nm is simulated. Fig. 2 show the variation of gold (Au) permittivity, and different values of d_m as a function of wavelength. As we can see, the variation of real part (Fig. 2(a)) is neglected when the width of metal layer is changed, because the real part in (4) is partially depends on Γ^*. While the imaginary part, when the width of metal layer goes below 10nm, extremely changes.

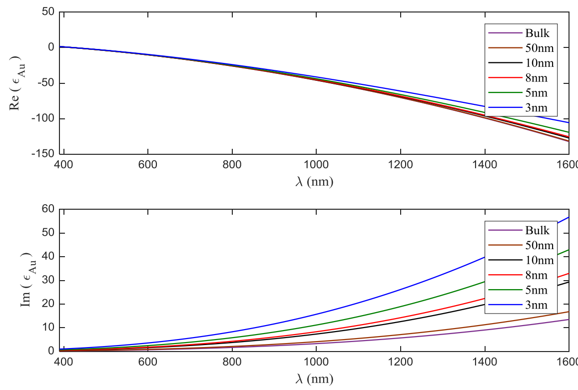

Fig. 2. Real and imaginary parts of permittivity for different values of d_m as a function of wavelength for Au.

The reduction of the geometry size below 10nm indicate the major decrease of mean free path of electrons in metal. This limitation effect on the transverse and perpendicular amplitude of permittivity. So, reducing the thickness of the metal layer from a certain amount can eliminate the hyperbolic property of the layers. To illustrate this important and to select the optimal thickness of the metal layer, the effective permittivity of the unit cell is simulated for different filling factor values of the metal in Fig. 3. Interestingly, the imaginary part of ε_{eff} shows a broadened resonance peak due to the electromagnetic interactions between the metal and dielectric. The resonance band in ε_{eff} is very broad in the curve where $f_m = 0.4$.

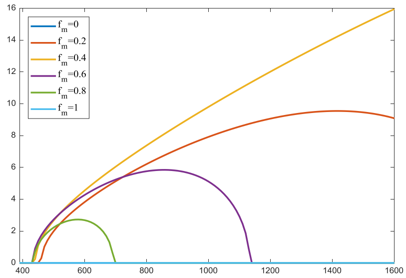

Fig. 3. Calculation of the imaginary parts of the effective permittivity of unit cell in Fig. 1 for different values of filling factor of the metal.

Considering a slab composed of 15 unit cells illuminated by an incident TM polarized CW probe whose wavelength varies in the visible range and using (5) -(7). We have calculated the real and imaginary parts of effective permittivity for two cases. In the first case, we ignored the effect of thickness of metal and used constant Γ in the Drude model. Then by (5) and (6) calculate the effective permittivity. The solid curves in fig. 4 illustrate the results of simulation. After that, we used (3) and taking the effect of thin metals into account. As we can see from fig. 4, the magnitude of longitudinal permittivity extremely decreased. Real and imaginary parts of $\varepsilon_{\perp}$ decreased about 42.58% and 52.68% respectively. The magnitude diminution has bounded the transmission and reflection proportion.

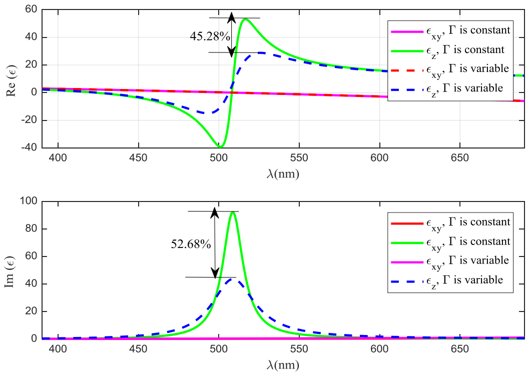

Fig. 4. Real and imaginary parts of the effective permittivity components vs. without and with Kerr effect.

Then we calculated transmission and reflection coefficients based on eq. (9). In order to manifest the accuracy of results, we have also calculated transmittance and reflectance curves theoretically. As shown in Fig. 5, the simulation results show the agreement between the theoretical and simulated is good. In the whole wavelength band, the deviation from this agreement is less than 1%.

4 Conclusion

In this paper, we present a new model to calculate the permittivity of ultra-thin metal. Then the effect of new model applied to HMM structures and effective permittivity and transmission and reflection coefficient are calculated. The results compared for two cases, constant Γ and variable Γ, to each other's.

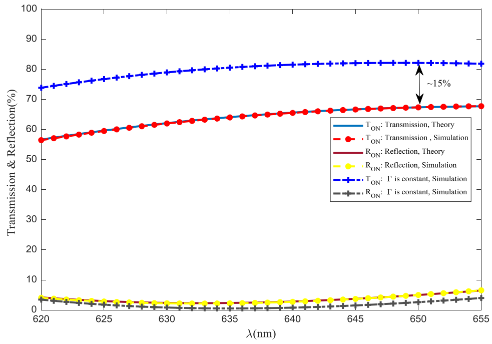

Fig. 5. Transmission and reflection coefficients for HMM.

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