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مدل مداری آرایه موجبری بر پایه تقارن پاریته-زمان ($\mathcal{PT}$)

الناز پیله ور[۱]، محمدکاظم مروج فرشی[۱] و حمیدرضا رمضانی[۲]

[۱]دانشگاه تربیت مدرس-دانشکده مهندسی برق و کامپیوتر-گروه الکترونیک-ایران

[۲] دانشگاه تگزاس، ریو گرند ولی (UTRGV)- گروه فیزیک-آمریکا

چکیده –یک نوع مدل مداری برای تعیین مشخصه مود (مقادیر ویژه) آرایه متناوب موجبر متقارن $\mathcal{PT}$ پیشنهاد شده است. این مدل شامل یک جفت مدار LC کوپل شده به هم است. یکی از این دو زوج دارای مقاومت $-R$ است و بهره تولید می‌کند و دیگری دارای تلف معادل آن است. ساختار نوار این آرایه موجبری دارای یک گاف است که هیچ مودی در آن منتشر نمی‌شود. اندازه این گاف به مقدار بهره/تلف (γ) بستگی دارد. به عبارت دیگر، با تغییر اندازه γ می توان مودهای مجاز به انتشار در آرایه‌ی $\mathcal{PT}$ را کنترل کرد. به ازای مقدار خاصی از $(\gamma=\gamma_{PT})$، که نقطه بحرانی نامیده می‌شود، مقدار گاف صفر می‌شود. نتایج مدل مداری با مدل اپتیکی آن مطابقت خوبی دارند. ویژگی مهم این طراحی سرعت بالا و کوک پذیری آن در عین سادگی مدار است.

کلید واژه- تقارن پاریته زمان ($\mathcal{PT}$)، موجبر نوری، مدل مداری RLC

Circuit Model of Parity-Time ($\mathcal{PT}$) Symmetric Waveguide Arrays

Elnaz Pilehvar[1], Mohammad Kazem Moravvej-Farshi[1], Hamidreza Ramezani[2]

Faculty of Electrical and Computer Engineering, Advanced Devices Simulation Lab (ADSL), Tarbiat Modares University, P. O. Box 14115-194, Tehran 1411713116, Iran.

[2] Department of Physics, The University of Texas Rio Grande Valley, Brownsville, TX 78520, USA

Abstract- A circuit model is proposed for determining the characteristic modes (eigenvalues) of a periodic array of PT-symmetric (gain/loss coupled) waveguides. This model consists of a pair of coupled LC circuits, one with amplification resistance -R and the other with an equivalent attenuation. The band structure consists of two bands separated by a gap that depends on the value of the gain/loss (γ). In other words, one can control the size of bandgap and hence the allowed modes propagating within the $\mathcal{PT}$ array. For a critical value of gain/loss or ($\gamma=\gamma_{PT}$), the eigenvalues merge and the gap disappears. The corresponding results are in good agreement with the optical model results. The significance of this design lies with its tunability, high speed, and relative simplicity.

Keywords: Parity-Time ($\mathcal{PT}$) Symmetry, Optical Waveguides, RLC Circuit Model.

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Elnaz Pilehvar **Mohammad Kazem Moravvej-Farshi** **Hamidreza Ramezani**

e.pilehvar@modares.ac.ir moravvej@modares.ac.ir hamidreza.ramezani@utrgv.edu

1 Introduction

Parity time ($\mathcal{PT}$) symmetry first started in quantum mechanics in 1998 [1]. In this regard, it was recognized that a new class of non-Hermitian Hamiltonians that commute with the $\mathcal{PT}$ operator — i.e., $[H\ \mathcal{PT}] = 0$ — may have an entirely real energy spectrum. The parity operator $\mathcal{P}$ ($r \to -r$) acts as a spatial reflection and the time reversal operator $\mathcal{T}$ ($p \to -p; r \to r; i \to -i$) acts as a time reflection. Making such a condition requires that the real part of the potential should be symmetric while, the imaginary part is anti-symmetric. Therefore it is necessary to have a complex potential which isn't possible in reality. Thanks to the similarities between the Schrödinger equation and paraxial equation of diffraction that describes the dynamics of light. We can carry over the principal from quantum mechanics (QM) to optics. They are similar in equations such that, a $\mathcal{PT}$ potential in QM can be replaced by a $\mathcal{PT}$ refractive index in optics that demands a complex refractive index, $n(r)$, obeying $n(r) = n^*(-r)$. In other words, the $\mathrm{Re}[n(r)] = \mathrm{Re}[n(-r)]$ is an even function of coordinate *r*, while $\mathrm{Im}[n(r)] = -\mathrm{Im}[n(-r)]$ is an odd function, which determines the size of gain/ loss parameter (γ) [1-6].

A $\mathcal{PT}$ optical medium can be implemented by a pair of coupled gain/loss waveguides or a $\mathcal{PT}$ dimer. In other words, when one of the two paired waveguides is made of a material with a pure loss of known amplitude, its coupled counterpart should be made of a material with a pure gain of the same amplitude [3]. Considering a $\mathcal{PT}$ dimer, with gain/loss below the critical value $(\gamma < \gamma_{\mathrm{PT}})$, a supermode that is a field almost equally distributed between the gain and loss waveguide is formed. Hence, the imaginary part of the eigenvalue stays around the zero. This regime is called the exact phase or the $\mathcal{PT}$-symmetric regime. Otherwise, the spontaneous $\mathcal{PT}$ symmetry breaking occurs for $\gamma = \gamma_{\mathrm{PT}}$ that is known as the exceptional point or the non-Hermitian degeneracy; and the eigenvalues become complex for $\gamma > \gamma_{\mathrm{PT}}$ that is called the broken phase regime, in which an electric field becomes squeezed mainly in the gain or loss waveguide [1-6].

In this paper, we have proposed a circuit model for a periodic array of $\mathcal{PT}$ dimers next to each other, to determine the propagation eigenvalue diagram that consists of two bands separated with a gap in between. The proposed circuit model consists of two coupled LRC oscillators, one with $-R$ acting as the gain material in each dimer and the other with R for loss material. By varying the γ value, one can control the bandgap region of the $\mathcal{PT}$ array or the ranges of the allowed modes propagate within the array. This allows a direct observation of a phase transition from a real to a complex eigenfrequency spectrum.

2 The $\mathcal{PT}$ Array Optical Model

Figure 1 illustrates a 3D schematic of the $\mathcal{PT}$ array, under study, consisting *N* coupled unit cells (dimers), each composed of a gain waveguide (red) coupled to a loss waveguide (green). Each waveguide is assumed to be 500 nm×237 nm InGaAsP rod grown on an InP substrate and is single-mode at the operation wavelength of $\lambda_0 = 1.55\,\mu\mathrm{m}$. The intra- and inter-dimer coupling coefficients are K_1 and K_2, respectively, and assumed to be $K_1 \approx K_2 > 0$. Using the coupled

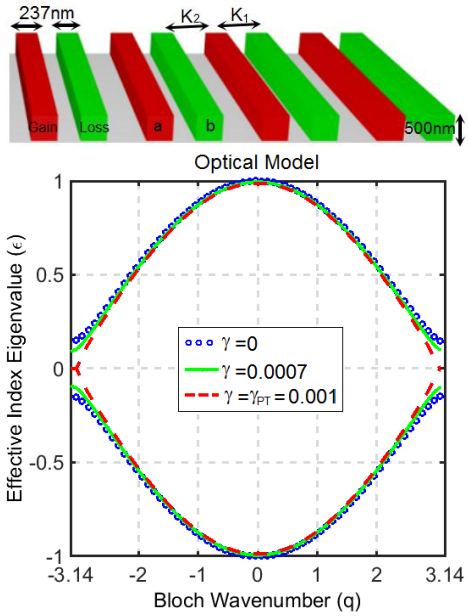

Figure 1: (a) $\mathcal{PT}$ array structure with intra- and inter-dimer coupling K_1 and K_2. Sublattices cuboid of a and b are gain and loss materials. Dimensions are designed in such a way that all of the waveguides are single mode. (b) Normalized Dispersion diagram for various γ values.

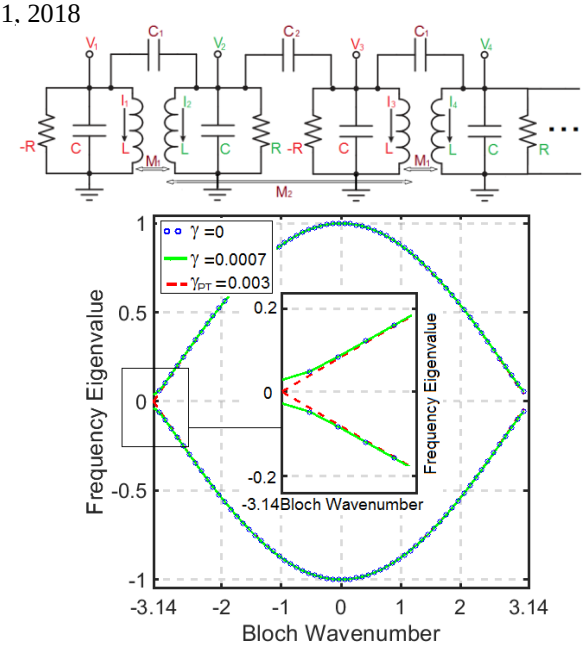

Figure 2: (a) The circuit model of the structure consists of a coupled LRC oscillator, one with $-R$ for gain material and the other with R for loss material. Mutual inductances (M_1, M_2) and capacitive (C_1, C_2) coupling are included for generality. (b) Normalized dispersion diagram for various γ values. At $\gamma = \gamma_{PT}$ the gap between the two bands disappears. The inset shows a zoomed-in portion of the figure about the zone edge.

mode theory, one can easily obtain the band structure for this diatomic $\mathcal{PT}$ array [4,7]:

$$\varepsilon_{\pm}(\mathrm{q}) = \pm\sqrt{K_1^2 + K_2^2 + 2K_1K_2\cos(q) - \gamma^2} \qquad (1)$$

where ε and q representing the effective refractive index and Bloch wavenumber. Figure 1(b) illustrates the plots of the eigenvalues ($\varepsilon_{\pm}$) versus q, for three different values of $\gamma = 0$ (no loss/no gain), 0.0007 (exact phase regime), and 0.001 (exceptional point). As can be observed from the figure, in the exact phase regime, the band structure spectrum has two branches separated by a gap whose maximum value of $2(K_1 - K_2)$ occurs for $\gamma = 0$ at the Brillouin zone edges ($q = \pm\pi$).The gap decreases as γ increases until it disappears at the exceptional point for which $\gamma = K_1 - K_2 = \gamma_{\mathrm{PT}}$. In other words, at the exceptional point, the real eigenvalue becomes 2-fold degenerate for $q = \pm\pi$ and the $\mathcal{PT}$ symmetry breaks down. Beyond this point, however, the eigenvalues are no longer real that is known as the broken phase regime.

3 Circuit Model of $\mathcal{PT}$ Array

When the wavelength is greater than the dimensions of the system, we can model all spatial symmetry considerations in the form of a network defined the Kirchhoff's law [8-9]. Figure 2(a) shows the simple circuit model for the proposed $\mathcal{PT}$-symmetric array. This model consists of a pair of coupled LC circuits, one with amplification resistance $-R$ and the other with equivalent attenuation which inductances and capacitances (M_1, C_1) for intra and (M_2, C_2) for inter-coupling are used between the oscillators which results in the gain/loss parameter $\gamma = R^{-1}\sqrt{L/C}$. A 2D field solver is usually used to extract the parameters for a given geometry and an LC matrix can be set up. By applying the Kirchhoff's law for each arbitrary node, we have:

$$\begin{cases} V_j = i\omega L I_j + i\omega M_{j-1} I_{j-1} + i\omega M_j I_{j+1} \\ I_j + \dfrac{V_j}{R} + i\omega C V_j + i\omega C_{j-1}(V_j - V_{j-1}) + i\omega C_j (V_j - V_{j+1}) = 0 \end{cases} \qquad (2)$$

where V_j and I_j are the voltage dropped across the $j-th$ inductor and the current passing through it. Both mutual capacitive and inductance coupling are included for generality, although in the experimental results exclusively one or the other is

accounted [8]. By expanding the relations to the node, scaling the frequency by $\omega_0 = 1/\sqrt{LC}$, ignoring M_1, M_2 and eliminating currents from the relations, we have:

$$\begin{bmatrix} F_1 - i\gamma & F_2 & 0 & 0 & \dots & 0 \\ F_2 & F_1 - F_3 + i\gamma & F_3 & 0 & \dots & 0 \\ 0 & F_3 & F_1 - F_3 - i\gamma & F_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & F_2 & F_1 + i\gamma \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \dots \\ \dots \\ V_N \end{bmatrix} = 0 \quad (3)$$

$$F_1 = \frac{1}{\omega} - \omega(1 + C_1/C)\ ,\ F_2 = \omega C_1/C\ ,\ F_3 = \omega C_2/C$$

This linear homogeneous system has N normal mode frequencies which N equals to the number of dimers in the system. For a large N, the eigenvalues are closely spaced and the band diagram is obtained more accurately. Otherwise, the eigenvalues are further apart. We should consider the total number of waveguides to be rather large but finite. The values of q spaced by $2\pi/N$ and run from $-\pi$ to $+\pi$. Two factors restrict the number of eigenvalues: firstly, finite waveguides can have only a finite number of eigenvalues. Secondly, q values which differing by 2π, specify same states on a discrete lattice and only the values of q within a range of 2π yields independent solution [10]. As seen in Figure 2(b) for $N = 20$, in the exact phase not close to $(\gamma = \gamma_{PT})$, the imaginary frequency component is then zero and there is a gap region between two bands. In critical value of gain/loss or $(\gamma = \gamma_{PT})$ the eigenfrequencies coalesce together and the gap will be closed. In fact, by changing the γ value, one can control the bandgap region of a $\mathcal{PT}$ array or the ranges of modes that are allowed to propagate in it. A comparison with results of the optical model in Figure 1 indicated good agreement which was obtained through a circuit model of Figure 2 and the difference could be due to ignoring some parameters such as inductance couplings, approximation in calculating parameters and etc.

4 Conclusion

$\mathcal{PT}$-symmetric electronic circuit model opens a novel functionality in the spatiotemporal domain. An approximate circuit model with an oscillator is proposed for the mode management in a $\mathcal{PT}$ array which is required for applications such as in optical communications systems and nonlinear optical systems. By tuning the gain/loss value or γ, one can control the bandgap in the eigenvalue diagram of a $\mathcal{PT}$ array or select the desired mode and suppress other modes. The important feature of this design is high-speed tunability with the simple practical circuit.

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