Abstract- In this paper, the sensitivity (S) and figure of merit (FOM) of disk resonator based photonic biosensors as a function of the gap distance, are studied. These biosensors include a whispering gallery mode (WGM) silicon disk resonator (7µm external radius), coupled to a waveguide (500 nm wide). Gap distance is changed in the range of 150-350 nm. The target is to find the optimized parameters of biosensor device for obtaining high S and FOM. Calculations have been done based on the analytical conformal transformation method (CTM), and coupled mode theory (CMT). Our fully analytical methods have an effective role in fast design of WGM resonator based biosensors. Based on our calculations, the S and FOM parameters of optimized biosensor can be increased up to 134 nm/RIU and 2200 /RIU, at resonance wavelength of 1550 nm. The high FOM and the small size radius of this disk resonator biosensor, allow a high performance device with application in sensing of low amounts of analyte.

Keywords: Photonic biosensor, Whispering gallery mode, Sensitivity
1 Introduction

Photonic biosensors are an important group of devices with numerous applications in the food and biomedical industry including gas sensing, glucose measurement, pathogen detection, and the study of protein–protein interactions. The photonic biosensors based on surface-plasmon-resonance (SPR), [1], has become the most common commercial implementation of evanescent-wave sensors [2]. However, there is a great deal of attention into alternative evanescent-wave based sensors. These kind of sensors could provide improvements in sensitivity, robustness, device size, and easy integration with optical sources and detectors [3]. Among the evanescent wave base biosensors, Whispering gallery mode (WGM) resonator based sensors are especially gaining attention. These biosensors could show high amount of quality factor (up to 10^{11}) and narrow resonances (causes low amounts of limit of detection) [4]. In WGM resonator based photonic biosensors, evanescent-field part of electromagnetic resonance, interact with the biological samples (analyt) on the medium. If the analyts have homogeneously distributed in the biological solution, bulk sensitivity (S) can be measured [3]. Here we examine the effect of gap distance changes on S and figure of merit (FOM) of a photonic biosensor based on whispering gallery mode disk resonator. Disk resonator utilization has the advantage of decreased scattering loss, compared to a ring resonator (since there is only one edge from which light can scatter). Furthermore a disk resonator can support multi WGM propagation. Regard this property, they can be used in biosensor devices with multi-analyte detection. Using the optimum gap distance between resonator and waveguide, maximize S and FOM (S=134 nm/RIU and FOM= 2200/RIU) will be demonstrated for transverse electric (TE) disk resonator based biosensors.

2 Principle and design

A schematic of considered biosensor is presented in Fig 1.

The biosensor device consists of a WGM disk resonator laterally coupled to a straight waveguide. The input signal is directed to micro-disk. As it can be seen in Fig.1, the evanescent field of WGM resonance can interacts with biological solution. This solution including analyts, exist in close contact of resonator surface. These devices have been designed for TE modes. The core material is silicon with refractive index of n_{core} =3.45. Both the substrate and cladding materials are silicon dioxide with n_{sub}= n_{cl}=1.46. The width of the bus waveguide (2w_1), has been fixed to 500 nm. Equivalent resonator waveguide width (2w_2) for disk resonator, is calculated as [5]. The gap distance between the straight waveguide and the WGM resonator is changed in range of 150-350 nm. The curvature radius has been imposed to R≥7μm, in order to have negligible bending losses.

2.1 Problem formulation

When evanescent field part of a WGM (as a sensing signal), interact with analyt molecules, effective refractive index (n_{eff}) of WGM resonance wavelength, is changed. In fact, it causes a change of the effective refractive index (Δn_{eff}) of the optical confined WGM. This produces a net spectral shift in the resonator WGM resonance wavelength (Δλ_{WGM}). Δλ_{WGM} is related to Δn_{eff} by the well-established resonance condition:

\[ \Delta \lambda_{WGM} = \frac{2\pi R}{m} \Delta n_{eff} \]

where R is the WGM resonator radius and m is an integer representing the number of optical wavelengths around the resonator perimeter. By measuring Δλ_{WGM} caused by Δn_{eff}, the analyte detection becomes possible. So first, we must be able to calculate exact amount of n_{eff} experienced by WGM resonance wavelength in both the air and biological claddings.

2.2 Effective refractive index

For effective refractive index (n_{eff}) calculation, we use an analytical method. n_{eff} of WGM resonance wavelength in disk resonator have been found by conformal transformation (CTM), method [6]. In this method, a curved waveguide in real space is replaced with a straight waveguide in complex space. For finding n_{eff} of fundamental mode in straight waveguide, we have used a graphical method named b-v diagrams [7].
3 Result and discussion

3.1 Quality factor

After calculation of resonance mode $n_{eff}$, we can find the coupling coefficient ($\kappa$) between disk resonator and straight waveguide. Then, coupling quality factor ($Q_k$) of the resonance modes, can be obtained. A two dimensional coupled mod theory (2D CMT) [8], has been used for coupling analysis of the system. For considered disk resonator, intrinsic quality factor ($Q_i$) is found to be $Q_i \geq 20000$. Finally total quality factor which expressed as $Q_k^{-1} = Q_i^{-1} + Q_s^{-1}$, can be calculated. We examine variation of mentioned parameters in gap distance range of 150-350 nm. Since the considered disk resonators can support multi-WGM propagation. All the calculations and graphs have been found for two first WGM resonance wavelength. The calculation results have been shown in Figs 2-4.

$$Q_{fs} = 3.970 \times 10^6$$

3.2 Sensitivity

The homogenous or bulk refractive index sensitivity of a WGM photonic based biosensor is defined as: $S=\Delta \lambda_{WGM}/\Delta n_{sol}$.Where $\Delta \lambda_{WGM}$ is calculated with using Eq.1, $\Delta n_{sol}$ is bulk refractive index change of the solution flowing on top of the micro-disk. For $S$ calculation, $\Delta n_{eff}$ and $\Delta \lambda_{WGM}$ are calculated in different concentrations of glucose/water solutions (0.0 to 9.0% weight/weight) flowing on cladding layer. To simulate these concentrations, we have changed the cladding refractive index from 1.333 to 1.350 (according to empirical relation of $\Delta n_{sol} = C x 1.375 \times 10^{-3}$ RIU/% [9], [10], between glucose concentration $C$ and $\Delta n_{sol}$).

![Fig. 3. Sensitivity of WGM disk resonator based biosensor with refractive index variation of solution.](image)

3.3 Figure of merit

One of the characterizing parameters in biosensing domain is FOM. FOM depends only on the characteristics of the transducer part of an optical biosensor [11]. In case of a WGM based biosensor with total quality factor of $Q_t$, FOM can be defined as $\text{FOM} = Q_t/\lambda_{WGM}$. Variation of FOM in different ranges of gap distance, has been presented in Fig. 4.

![Fig. 4. Variations of FOM in different ranges of gap.](image)
Also another parameters in bio-sensing domain, is intrinsic limit of detection (ILOD). It is defined as 
\[ ILOD = FOM^{-1} = \frac{\lambda_{WGM}}{Q \times S} \] [12]. ILOD is the numerical amount of minimum refractive index 
unit change (\[ \Delta n_{\text{min}}=\text{ILOD} \]), that can be detected by 
the resonator. Both the FOM and ILOD parameters, allow us to compare performance of 
photon biosensors with different sensing mechanisms. Our proposed disk resonator based 
biosensor, shows the ILOD of \[ \leq 4.54 \times 10^{-4} \text{RIU} \].

4 Conclusion

In the obtained optimized WGM disk resonator 
based biosensor, WGM with resonance wavelength 
of \[ \lambda \approx 1550 \text{ nm} \], is found to exhibit most S and FOM 
of 134 nm/RIU and 2200/RIU (at gap distance of 
350 nm). Based on the high calculated Q, values of 
this resonance at this resonator, in water solutions, 
ILOD of approximately as \[ \leq 4.5 \times 10^{-4} \text{RIU} \] for disk 
resonator sensor, was obtained. This amount of 
ILODs, have been improved by a factor of 0.6 
compare with an SOI optimized strip waveguide 
resonator sensor with ILOD= 7.5 \times 10^{-4} \text{RIU} \) [13]. 
A final remark is that we have found the optimized parameters for \[ \lambda \approx 1550 \text{ nm} \], commonly used for 
telecommunications. Therefore, it has been very 
well characterized and offers several low-cost 
components. This study will allow creation of a 
fast and reliable biosensor, based on photonic disk 
resonator with high performances. These 
biosensors have applications in the areas of 
refractive index-based medical diagnoses and life 
sciences.

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