Abstract- In this paper, the sensitivity (S) and figure of merit (FOM) of disk resonator based photonic biosensors as a function of the gap distance, are studied. These biosensors include a whispering gallery mode (WGM) silicon disk resonator (7µm external radius), coupled to a waveguide (500 nm wide). Gap distance is changed in the range of 150-350 nm. The target is to find the optimized parameters of biosensor device for obtaining high S and FOM. Calculations have been done based on the analytical conformal transformation method (CTM), and coupled mode theory (CMT). Our fully analytical methods have an effective role in fast design of WGM resonator based biosensors. Based on our calculations, the S and FOM parameters of optimized biosensor can be increased up to 134 nm/RIU and 2200 /RIU, at resonance wavelength of 1550 nm. The high FOM and the small size radius of this disk resonator biosensor, allow a high performance device with application in sensing of low amounts of analyte.

Keywords: Photonic biosensor, Whispering gallery mode, Sensitivity
1 Introduction

Photonic biosensors are an important group of devices with numerous applications in the food and biomedical industry including gas sensing, glucose measurement, pathogen detection, and the study of protein–protein interactions. The photonic biosensors based on surface-plasmon-resonance (SPR), [1], has become the most common commercial implementation of evanescent-wave sensors [2]. However, there is a great deal of attention into alternative evanescent-wave based sensors. These kind of sensors could provide improvements in sensitivity, robustness, device size, and easy integration with optical sources and detectors [3]. Among the evanescent wave base biosensors, Whispering gallery mode (WGM) resonator based sensors are especially gaining attention. These biosensors could show high amount of quality factor (up to 10^{11}) and narrow resonances (causes low amounts of limit of detection) [4]. In WGM resonator based photonic biosensors, evanescent-field part of electromagnetic resonance, interact with the biological samples (analyt) on the medium. If the analyts have homogeneously distributed in the biological solution, bulk sensitivity (S) can be measured [3]. Here we examine the effect of gap distance changes on S and figure of merit (FOM) of a photonic biosensor based on whispering gallery mode disk resonator. Disk resonator utilization has the advantage of decreased scattering loss, compared to a ring resonator (since there is only one edge from which light can scatter). Furthermore a disk resonator can support multi WGM propagation. Regard this property, they can be used in biosensor devices with multi-analyte detection. Using the optimum gap distance between resonator and waveguide, maximize S and FOM (S=134 nm/RIU and FOM= 2200/RIU) will be demonstrated for transverse electric (TE) disk resonator based biosensors.

2 Principle and design

A schematic of considered biosensor is presented in Fig 1.

![Figure 1: A photonic WGM disk resonator based biosensor](image)

The biosensor device consists of a WGM disk resonator laterally coupled to a straight waveguide. The input signal is directed to micro-disk. As it can be seen in Fig.1, the evanescent field of WGM resonance can interacts with biological solution. This solution including analyts, exist in close contact of resonator surface. These devices have been designed for TE modes. The core material is silicon with refractive index of n_{eff} =3.45. Both the substrate and cladding materials are silicon dioxide with n_{sub} = n_{cla}=1.46. The width of the bus waveguide (2w_1), has been fixed to 500 nm. Equivalent resonator waveguide width (2w_2) for disk resonator, is calculated as [5]. The gap distance between the straight waveguide and the WGM resonator is changed in range of 150-350 nm. The curvature radius has been imposed to R=7μm, in order to have negligible bending losses.

2.1 Problem formulation

When evanescent field part of a WGM (as a sensing signal), interact with analyt molecules, effective refractive index (n_{eff}) of WGM resonance wavelength, is changed. In fact, it causes a change of the effective refractive index (Δn_{eff}) of the optical confined WGM. This produces a net spectral shift in the resonator WGM resonance wavelength (Δλ_{WGM}). Δλ_{WGM} is related to Δn_{eff} by the well-established resonance condition:

\[
Δλ_{WGM} = \frac{2πR}{m}Δn_{eff}
\]

where R is the WGM resonator radius and m is an integer representing the number of optical wavelengths around the resonator perimeter. By measuring Δλ_{WGM} caused by Δn_{eff}, the analyte detection becomes possible. So first, we must be able to calculate exact amount of n_{eff} experienced by WGM resonance wavelength in both the air and biological claddings.

2.2 Effective refractive index

For effective refractive index (n_{eff}) calculation, we use an analytical method. n_{eff} of WGM resonance wavelength in disk resonator have been found by conformal transformation (CTM), method [6]. In this method, a curved waveguide in real space is replaced with a straight waveguide in complex space. For finding n_{eff} of fundamental mode in straight waveguide, we have used a graphical method named b-v diagrams [7].
3 Result and discussion

3.1. Quality factor

After calculation of resonance mode $n_{eff}$, we can find the coupling coefficient ($\kappa$) between disk resonator and straight waveguide. Then, coupling quality factor ($Q_k$) of the resonance modes, can be obtained. A two dimensional coupled mode theory (2D CMT) [8], has been used for coupling analysis of the system. For considered disk resonator, intrinsic quality factor ($Q_i$) is found to be $Q_i \geq 20000$. Finally total quality factor which expressed as $Q_t^{-1} = Q_k^{-1} + Q_i^{-1}$, can be calculated. We examine variation of mentioned parameters in gap distance range of 150-350 nm. Since the considered disk resonators can support multi-WGM propagation. All the calculations and graphs have been found for two first WGM resonance wavelength. The calculation results have been shown in Figs 2-4.

3.2. Sensitivity

The homogenous or bulk refractive index sensitivity of a WGM photonic based biosensor is defined as: $S = \Delta \lambda_{WGM}/\Delta n_{sol}$. Where $\Delta \lambda_{WGM}$ is calculated with using Eq.1, $\Delta n_{sol}$ is bulk refractive index change of the solution flowing on top of the micro-disk. For S calculation, $\Delta n_{sol}$ and $\Delta \lambda_{WGM}$ are calculated in different concentrations of glucose/water solutions (0.0 to 9.0 % weight/weight) flowing on cladding layer. To simulate these concentrations, we have changed the cladding refractive index from 1.333 to 1.350 (according to empirical relation of $\Delta n_{sol} = C \times 1.375 \times 10^{-3}$ RIU/%) [9], [10], between glucose concentration C and $\Delta n_{sol}$).

![Fig. 3. Sensitivity of WGM disk resonator based biosensor with refractive index variation of solution.](image)

3.3. Figure of merit

One of the characterizing parameters in biosensing domain is FOM. FOM depends only on the characteristics of the transducer part of an optical biosensor [11]. In case of a WGM based biosensor with total quality factor of $Q_t$, FOM can be defined as $FOM = Q_t/\lambda_{WGM}$. Variation of FOM in different ranges of gap distance, has been presented in Fig. 4.

![Fig. 4. Variations of FOM in different ranges of gap.](image)
Also another parameters in bio-sensing domain, is intrinsic limit of detection (ILOD). It is defined as 
\[ \text{ILOD} = \text{FOM}^{-1} = \frac{\lambda_{\text{WGM}}}{Q \times S} \] [12]. ILOD is the numerical amount of minimum refractive index 
unit change (\( \Delta n_{\text{min}} \)), that can be detected by the resonator. Both the FOM and ILOD 
parameters, allow us to compare performance of photonic biosensors with different sensing 
mechanisms. Our proposed disk resonator based biosensor, shows the ILOD of \( \leq 4.54 \times 10^{-4} \) RIU.

4 Conclusion

In the obtained optimized WGM disk resonator 

Based biosensor, WGM with resonance wavelength 

of \( \cong 1550 \) nm, is found to exhibit most S and FOM 
of 134 nm/RIU and 2200/RIU (at gap distance of 

350 nm). Based on the high calculated Q, values of 

this resonance at this resonator, in water solutions, 

ILOD of approximately as \( \cong 4.5 \times 10^{-4} \) RIU for disk 

resonator sensor, was obtained. This amount of 

ILODs, have been improved by a factor of 0.6 

compare with an SOI optimized strip waveguide 

resonator sensor with ILOD= 7.5 \times 10^{-4} \text{RIU} \] [13]. 

A final remark is that we have found the optimized parameters for \( \lambda \cong 1550 \) nm, commonly used for 
telecommunications. Therefore, it has been very 

well characterized and offers several low-cost 
components. This study will allow creation of a 

fast and reliable biosensor, based on photonic disk 

resonator with high performances. These 

biosensors have applications in the areas of 

refractive index-based medical diagnoses and life sciences.

References


Label-free detection with high-Q 

microcavities: a review of biosensing 

mechanisms for integrated devices." 


Photonics, Spectroscopy, and Microscopy. 


resonant sensors for biochemical applications." 


microresonators: fundamentals and applications." Rivista del Nuovo Cimento 34.7 


wave coupling of whispering gallery modes of 

dielectric cylinders." IEE Proceedings J 

(Optoelectronics) 140.3 (1993): 177-188.

curved optical waveguides by conformal 

transformation," IEEE Journal of Quantum 


rules for thin-film optical waveguides," 


optical routers based on Coupled Resonator 

Induced Transparency resonances." Optics 

[10] bin Mat Yunus, W. Mahmood, and Azizan 

bin Abdul Rahman. "Refractive index 
of solutions at high concentrations." Applied 


performance quantification of resonant 

refractive index sensors," Optics express, 16.2 


photonic micro-disk resonators for label-free 

biosensing." Optics express 21.7 (2013): 7994- 

8006.

sensitivity of Silicon-on-Insulator (SOI) strip 

waveguide resonator sensor." Biomedical 