



## ضریب جفت‌شدگی یک نانومزدوج موازی پلاسمونیک بر اساس میدان محوشونده

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چکیده - در این مقاله به محاسبه‌ی ضریب جفت‌شدگی در یک مزدوج موازی پلاسمونیک متشکل از دو موجبر تک‌لایه به فاصله‌هایی در محدوده‌ی ۲۰۰ نانومتر برای مد  $TM$  می‌پردازیم و فرض می‌کنیم که هر موجبر مانند یک اختلال برای موجبر دیگر عمل کند، ولی بر مدهای موجبر اثر نداشته باشد. ضریب جفت‌شدگی و فاصله‌ی انتقال بر حسب فرکانس بهنجار شده، در دو راستای  $x$  و  $z$ ، محاسبه و شبیه‌سازی شده و مقادیری در محدوده‌ی میلی‌متر و کمتر در یک پهنای فرکانسی به دست آمده است. بنابراین مناسب برای ساخت مدارهای مجتمع نوری و طراحی کوپلرها و کلیدهای نوری است.

کلید واژه- پلاسمونیک، ضریب جفت‌شدگی، فاصله‌ی انتقال، مدار مجتمع نوری

## Coupling Coefficient of Nano Plasmonic Directional Coupler Based on Evanescence Field

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Abstract- In this paper we obtain the coupling coefficient of plasmonic directional coupler (PDC) made up of two monolayer waveguides separated in the range of 200 nm for TM mode and we assume each waveguides acts as a perturbation to other waveguide but does not affect the waveguide mode. We numerically calculated the Transfer distance respect to normalized frequency and simulated in two directions,  $x$  and  $z$ , values are about millimetres and smaller in a limitation of a bandwidth. Thus it is suitable for designing of integrated optical circuits and construction of couplers and switches.

Keywords: plasmonic, coupling coefficient, transfer distance, integrated optical circuits

# Coupling Coefficient of Nano Plasmonic Directional Coupler Based on Evanescence Field

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## 1 Introduction

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Recent advances in the study of nanophotonics imply a fast developing and new promising field of science and technology which named plasmonics for exploiting optical features of metallic nano structures. Such material systems have shown their potential to spatially confine and manipulate light in subwavelength scale because of highly matched surface plasmon polaritons (SPPs), a mixed wave of light and free electron oscillation of conductors, actually on the metal surface [1].

Besides the applications of plasmonic devices in medical and biological fields such as using surface-enhanced Raman spectroscopy, the amazing aspect of plasmonic devices, confining light, makes such devices to have versatile applications for designing various type of devices i.e. different components of plasmonic circuits such as waveguides, couplers, switches and so on [2,3].

Because of the profound penetration of field into the metals and the material loss, using plasmonic waveguides is not that good for propagation in long-range. On the other hand they are good candidate for integrated optical circuit application in small scale regimes [4].

In this research initially we obtain the coupling coefficient of plasmonic directional coupler (PDC) which made up of two parallel plasmonic waveguide separated by a distance. We will use the evanescence field of two waveguides that induced polarization to obtain coupling coefficient. Transfer distance will be calculated numerically respect to normalized frequency and simulated for both components.

## 2 Equations

Monolayer plasmonic waveguide is one of the simplest waveguide that actually is an interface between a metal and dielectric [5]. In this paper, we incorporate two monolayer plasmonic waveguides which separate each other and we

assume each waveguides act as a perturbation to other waveguide but do not affect the other waveguide mode. Figure 1 illustrate a directional coupler that made of two separate monolayer plasmonic waveguide by a distance  $2a$  and filled by dielectric  $\varepsilon$ . The permittivity of metallic part of plasmonic waveguide is demonstrated as  $\varepsilon_c$ .

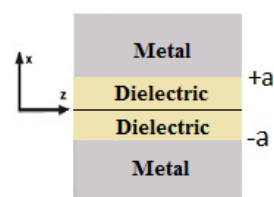


Figure 1: nano plasmonic directional coupler made of two monolayer plasmonic waveguides

Monolayer plasmonic waveguide only supports TM modes because only these modes can excite plasmon polaritons by tangential component of its electric field. If we take the coordinate system as shown in figure 1, the components of electric fields of TM mode are given by  $E_x$  and  $E_z$ . We take the upper waveguide of PDC as number 1 and the lower one as number 2, in general the electric field of upper and lower waveguide of PDC by taking into account a weak interaction of two waveguides are written as following:

$$E_{x_1}(x, z) = A_1(z)F_1(x)\exp(i\beta z) \quad (1)$$

$$E_{z_1}(x, z) = B_1(z)G_1(x)\exp(i\beta z) \quad (2)$$

$$E_{x_2}(x, z) = A_2(z)F_2(x)\exp(i\beta z) \quad (3)$$

$$E_{z_2}(x, z) = B_2(z)G_2(x)\exp(i\beta z) \quad (4)$$

Where  $A$  and  $B$  are slowly varying amplitudes,  $F(x)$  and  $G(x)$  are normalized transvers functions of each waveguide alone respectively. Lower waveguide undergoes a perturbation of the medium for upper waveguide and vice versa. This perturbation is appearance as difference of refractive index  $n_c - n$ . By implying Helmholtz equations to electric fields one can write:

$$\nabla^2 E_{x_1} + k_c^2 E_{x_1} = -\mu_0 \omega^2 \varepsilon_0 \left[ (\varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega}) - \varepsilon \right] E_{x_2} \quad (5)$$

$$\nabla^2 E_{z_1} + k_c^2 E_{z_1} = -\mu_0 \omega^2 \varepsilon_0 \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] E_{z_2} \quad (6)$$

$$\nabla^2 E_{x_2} + k_c^2 E_{x_2} = -\mu_0 \omega^2 \varepsilon_0 \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] E_{x_1} \quad (7)$$

$$\nabla^2 E_{z_2} + k_c^2 E_{z_2} = -\mu_0 \omega^2 \varepsilon_0 \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] E_{z_1} \quad (8)$$

Where  $\varepsilon_0, \mu_0, \sigma, \omega$  are the vacuum electric permittivity, vacuum magnetic permeability, conductivity and frequency, respectively.  $k$  is the wave vector which is written for metals by a subscript c and

$$k_c^2 = \beta^2 - k_0^2 \varepsilon_c \quad (9)$$

$$k^2 = \beta^2 - k_0^2 \varepsilon \quad (10)$$

That  $\beta$  is propagation constant written as

$$\beta = \sqrt{\frac{\varepsilon \varepsilon_c}{\varepsilon + \varepsilon_c}} \quad (11)$$

We suppose that propagation constant and transverse field distribution is unchanged under the effect of this perturbation, only the amplitudes  $A_1(z), B_1(z), A_2(z)$  and  $B_2(z)$  are taken as function of  $z$  although for monolayer plasmonic waveguide alone are constant. If the slowly varying amplitude is impose to equation (5-8) four coupled first-order differential equations are given:

$$\frac{\partial A_1}{\partial z} = iC_{12} A_2 \quad (12)$$

$$\frac{\partial B_1}{\partial z} = iC_{12} B_2 \quad (13)$$

$$\frac{\partial A_2}{\partial z} = iC_{21} A_1 \quad (14)$$

$$\frac{\partial B_2}{\partial z} = iC_{21} B_1 \quad (15)$$

Parameter Cs are called coupling coefficients and defined as following:

$$C_{12_x} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{-\infty}^{-a} F_1^* F_2 dx \quad (16)$$

$$C_{12_z} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{-\infty}^{-a} G_1^* G_2 dx \quad (17)$$

$$C_{21_x} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{+a}^{+\infty} F_1^* F_2 dx \quad (18)$$

$$C_{21_z} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{+a}^{+\infty} G_1^* G_2 dx \quad (19)$$

The normalized transvers functions,  $F(x)$  and  $G(x)$  are written as the normalized field distribution of monolayer plasmonic waveguide [6].

Some calculation lead to the four coupling coefficients as follow:

$$C_{12_x} = C_{21_x} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{-\infty}^{-a} F_1^* F_2 dx = -i \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \frac{\sqrt{\varepsilon_c \varepsilon} \exp(-2ka)}{\varepsilon_c - \varepsilon} \frac{1}{k_c + k} \quad (20)$$

$$C_{12_z} = C_{21_z} = \frac{1}{2} \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \int_{-\infty}^{-a} G_1^* G_2 dx = -i \left[ \left( \varepsilon_c + \frac{i\sigma}{\varepsilon_0 \omega} \right) - \varepsilon \right] \frac{k_0^2}{\beta} \frac{\sqrt{\varepsilon_c \varepsilon k_c k} \exp(-2ka)}{\varepsilon_c - \varepsilon} \frac{1}{k_c + k} \quad (21)$$

Thus we have a PDC which has capability to couple the optical power from one waveguide into another. The Coupling length or transfer distance in both directions of  $x$  and  $z$  are calculated as

$$L_x = \frac{\pi}{2C_{12_x}}, \quad L_z = \frac{\pi}{2C_{12_z}} \quad (22)$$

According to (22), we simulated the transfer distance for both components by using Au as the metals and  $As_2S_3$  chalcogenide glass for dielectric layer. The results of simulations have been shown in figures 2 and 3. In each figure the horizontal axis is normalize frequency respect to plasma frequency  $\omega_p = 1.36 \times 10^{16} Hz$ .

Figure 2 shows the x component of real part of transfer distance and figure 3 shows the z component of it respect to normalize frequency [7]. The general behavior of  $L_z$  and  $L_x$  are the same initially decrease by increasing the frequency and then increase and in some interval of frequency the value of two transfer lengths are nearly constant and slightly change the flat interval by increasing the distance between two monolayer waveguide. On the other hand there is a huge difference between  $L_x$  and  $L_z$  because of their values, the value of  $L_x$  is in the order of kilometers while the order of  $L_z$  is under millimeters. We expected such result because the x component of electric field has a few contribution to induce polarization or current to other waveguide. So because of weak coupling a large transfer distance of  $L_z$ , it is not useful for designing integrated optical devices. On the other hand the values of  $L_z$  is small enough and it can be useful for designing switches or 3db couplers for integrated optical circuits manufacturing.

The minimum value of  $L_z$  is occurred at  $\omega / \omega_p = 0.165$  is equal to  $0.66 \mu m$  for  $2a=200nm$ .

The interesting result is the good flatness of transfer distance in the interval between  $\omega/\omega_p = 0.15$  and  $\omega/\omega_p = 0.175$ , this remarkable feature is made PDCs a good candidate for designing dispersion less photonic devices.

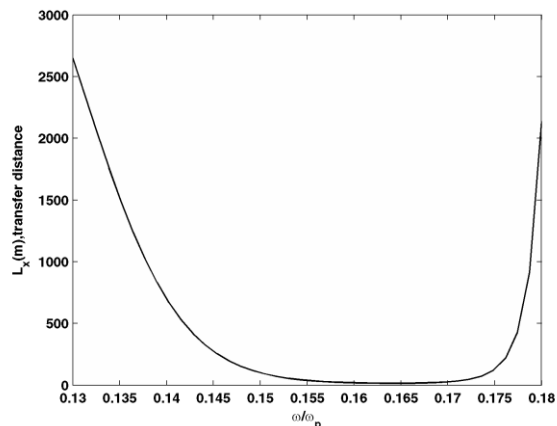


Figure 2: Real part of transfer distance as a function of normalized frequency in direction of x axis for nano plasmonic directional coupler at  $a=100$  nm.

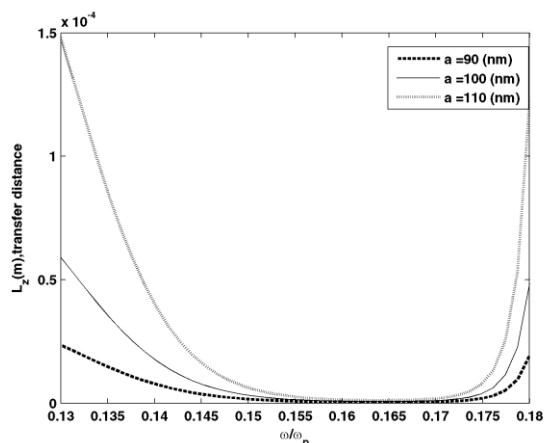


Figure 3: Real part of transfer distance as a function of normalized frequency in direction of z axis for nano plasmonic directional coupler at three distances  $a=90$  nm,  $a=100$  nm and  $a=110$  nm.

### 3 Conclusion

Confining light in plasmonic devices makes such devices to have versatile applications for designing various type of devices i.e. different components of plasmonic circuits such as waveguides, couplers, switches and so on.

In this research we obtained the coupling coefficient of plasmonic directional coupler (PDC). We calculated Transfer distance numerically respect to normalized frequency and simulated it for both components. The value of  $L_x$  which related to  $E_x$  is in the order of kilometers while the order of  $L_z$  related to  $E_z$  is under

millimeter. So because of weak coupling a large transfer distance of  $L_z$ , it is not useful for designing integrated optical devices. On the other hand the values of  $L_z$  is small enough and it can be useful for designing switches or 3db couplers for integrated optical circuits manufacturing. Also we found the optimum point of  $L_z$  and the band width in which  $L_z$  is almost flatness that is remarkable feature to make PDCs a good candidate for designing dispersion less photonic devices.

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