Abstract - In this paper, we investigate for the first time the parametric amplification in photonic crystal fibers (PCFs) with a continuously-decreasing zero-dispersion wavelength along their length. The use of these tapered fibers leads to the reduction of the length required for obtaining a high gain. Results show that although the gain is significantly increased, the gain bandwidth in such tapered PCFs is much narrower in comparison with that of un-tapered PCFs. Also, the gain flatness is quickly decreased with the increase of the input pump power.

Keywords: parametric amplification, photonic crystal fibers, dispersion-decreasing fiber, zero-dispersion wavelength.
Parametric amplification in dispersion-decreasing photonic crystal fibers

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1 Introduction

Parametric amplification relying on the four-wave mixing (FWM) has been widely studied in optical fibers [1, 2]. Nowadays, fiber optical parametric amplifiers (FOPAs) have found several applications in various areas such as high gain amplification, wavelength conversion, noise suppression, etc [3-6]. The fiber which is mostly used for amplifying telecommunication signal is the so-called highly nonlinear fiber (HNLF). An HNLF has a high nonlinear parameter, γ which is about 10 times higher than that of standard step-index optical fibers, with a zero-dispersion wavelength (ZDW) shifted in the telecom region, ~1550 nm [7]. Photonic crystal fibers (PCFs) which have a solid core and a cladding consisting of multiple air-holes that run along the length of the fiber core, exhibit even higher γ than HNLFs [8]. Dispersion characteristics of PCFs can be easily tailored for different wavelength regions by arranging the air-holes. In fact, by controlling the geometric parameters of PCFs such as the core diameter, the hole size and the pitch (hole-to-hole distance), it is possible to change γ and shift the ZDW to the desirable region. These properties make PCFs very suitable for nonlinear optics applications such as FWM, modulation instabilities, supercontinuum generation, etc [9].

The nonlinear length defined as

\[ L_{NL} = \frac{1}{\gamma P_0} , \]

determines the minimum fiber length required for the observation of nonlinear effects, where \( P_0 \) is the pump power which is input into the fiber [2]. A PCF can be tapered to achieve even smaller core than in an un-tapered PCF, thereby reducing the required fibre length and the pump power for observation of efficient nonlinear effects such as supercontinuum generation [10]. A recent review of tapered PCF fabrication, characterization, applications, and properties can be found in Ref. [11]. To the best of our knowledge, the parametric amplification has not been yet studied in the tapered PCFs with a ZDW decreasing along their length. In this article, we investigate parametric amplification in the ZDW decreasing PCFs and compare the results with those of un-tapered PCFs having a fixed ZDW. Results show that the tapered PCFs offer much higher parametric gain but with narrower bandwidth compared with those of the un-tapered PCF possessing the same length.

2 Theory

Based on the FWM, two pump photons at the angular frequency of \( \omega_p \) are degenerately combined at the fiber input with a signal photon at the angular frequency of \( \omega_s \) [2]. At the fiber output, in addition to the pump wave, the signal is amplified and a new wave, the so-called idler, is generated at angular frequency of \( \omega_i \). Three coupled equations which can be readily derived from the basic propagation equation are [5, 6]

\[ \frac{\partial A}{\partial z} = i \gamma \left[ |A_s|^2 + 2|A_p|^2 \right] A_i \]

\[ + 2 i \gamma A_s A_p^* \exp(i \Delta \beta z) - \frac{1}{2} \alpha A_i, \]  \hspace{1cm} (1)

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\[ + 2 i \gamma A_s A_p^* \exp(-i \Delta \beta z) - \frac{1}{2} \alpha A_i. \]  \hspace{1cm} (3)
where $A_p, A_s,$ and $A_i$ are the complex field amplitudes of the three co-polarized waves: pump, signal, and idler, respectively. The fiber loss is denoted by $\alpha$ and $\Delta\beta$ is the linear wave-vector mismatch of the interacting waves given by

$$\Delta\beta = -\frac{2\pi c}{\lambda_0^2} S_0 (\lambda_p - \lambda_0)(\lambda_p - \lambda_s)^2 \tag{4}$$

where $c$ is the light speed, and $\lambda_s$ and $\lambda_p$ are the signal and pump wavelengths, respectively. $\lambda_0$ is the zero-dispersion wavelength (ZDW) of the fiber and $S_0$ is the dispersion slope of the fiber calculated at $\lambda_0$. * denotes the complex conjugate.

Equations (1) - (3) can be solved numerically using the Runge-Kutta method to simulate the gain spectrum of the amplifier. For the un-tapered fibers, all the fiber parameters such as ZDW and $\gamma$ are considered to be fixed along the propagation distance $z$; however, these parameters for the tapered PCF have now $z$-dependence. Simulation is implemented by dividing the tapering length $0 \leq z \leq L$ into $N$ segments $z_1, z_2, ..., z_N$. Every time the fields move into a new segment $z_s$, the ZDW and $\gamma(z_s)$ are updated. Then, the parametric gain (in dB units) is calculated using the definition:

$$G = 10 \log \left( \frac{P_{s,\text{out}}}{P_{s,\text{in}}} \right), \text{ where } P_{s,\text{in}} = |A_s(0)|^2 \text{ and } P_{s,\text{out}} = |A_s(z = L)|^2 \text{ are the signal powers at the fiber input and output, respectively.}$$

### 3 Results and Discussion

In this section, we provide the simulation results for a tapered PCF and a pump source with parameters similar to those of in Ref. [10], and compare the results to those of un-tapered PCF. As it is shown in Fig. 1(a), the core diameter is exponentially decreased along the tapered PCF and simultaneously the nonlinear parameter $\gamma$ is increased (Fig. 1b), since we have $\gamma = \frac{n_2 \omega_p}{c A_{\text{eff}}}$. Here, $A_{\text{eff}}$ is the effective core area and $n_2$ is the nonlinear-index coefficient of silica. Also, as the core diameter decreases, the ZDW of the fiber is decreased exponentially as shown in Fig. 1(c).

Since an efficient FWM takes place when $\lambda_p \sim \lambda_0$, we assume that the ZDW at the fiber input $\lambda_0(z = 0) = 1063\text{nm}$ which is close to the wavelength of the Nd:YAG laser $\lambda_p = 1064\text{nm}$ with the power of $P_p = 2\text{W}$. The fiber loss is assumed to be 0.5 dB/km.
Fig. 3. Parametric gain spectrum at the end of the un-tapered PCF (a); and of the dispersion-decreasing PCF (b). The length of both PCFs is 12m and the input pump power is 2W.

Fig. 4. Parametric gain spectrum at the end of un-tapered PCF (a) and of the tapered PCF (b) for different input pump powers. For the dispersion-decreasing PCF, the smoothness of the gain spectrum is decreased with the pump power.

Fig. 2 shows the evolution of the gain spectrum along the tapered PCF. As it is seen, at the beginning of the fiber the gain is very low but its bandwidth is wide. This is because when the length is very small, the nonlinear interactions are small but since \( \lambda_\text{p} \sim \lambda_\text{ZDW} \), the bandwidth is wide. As the fiber length increases, its nonlinear parameter and in turn the nonlinear interaction increases, so the gain is increased. However, since the ZDW deviates from the pump wave the bandwidth is decreased. This is shown more explicitly in Fig. 3 where the output gain spectrum of an un-tapered PCF (Fig. 3a) with a fixed diameter of 5 \( \mu \)m is compared with that of the tapered PCF (Fig. 3b). As it is evident, the gain spectrum of the tapered PCF is much higher than that of the un-tapered PCF, but with much narrower bandwidth.

In Fig. 4, the effect of the input pump power on the flatness of the gain spectrum is depicted. As it is seen, the pump power significantly decreases the flatness of the gain spectrum of the tapered PCF.

4 Conclusion

We have investigated the parametric amplification in tapered PCFs with a decreasing ZDW along their length and compared the results to those of un-tapered PCFs. Since the nonlinear parameter substantially increases along the tapered PCFs, their gain is substantially increases. However as the ZDW is moved away from the pump wavelength, the gain bandwidth and its flatness are greatly decreased along the tapered PCFs.

References